

Number Theory by inquiry Old Westbury, NY

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What do we teach?

- Introduction to modular math based on students activity an inquiry

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- Guided by Instructor
- After students are comfortable with the defitions we begin our proof
- And sometimes we teach fun applications

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- The lessons are completely free - supported by donations and fees

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- About half of the students are female

Introduction to Number Theory

- Division

Introduction to Number Theory

- Division

We know how to do many things with these numbers: add, subtract, multiply and divide. However, when we divide we may write it differently

$$\begin{array}{r} 7 \text{ R } 2 \\ 3 \overline{)23} \end{array}$$

be written as $23 = 3 \cdot 7 + 2$. In general that a divided by b is written

$$a = b \cdot q + r$$

where q is called the **quotient** and r is the **remainder**. And $0 \leq r < b$. An important point here is that the q and r are unique. This fact is important

Introduction to Number Theory

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- Basic computations

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$$13 + 14 \pmod{4}$$

$$7 \cdot 21 \pmod{8}$$

$$7^{21} \pmod{8}$$

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$$3^{21} \not\equiv 11^{29} \pmod{7}$$

Definitions

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- What is division
- What is equivalence mod n

Definition. We say d divides n if $d = kn$ for some $k \in \mathbb{Z}$.

We write $3|12$ and we write $3 \nmid 13$. If $d|n$ we say

Proposition 2.1. We say $a \equiv b \pmod{n}$ if and only if $n|(b - a)$.

Some simple examples of modular math:

$$2 \equiv 7 \pmod{5} \text{ since } 5|(7 - 2)$$

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If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then \dots

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- Why do some conjectures succeed and some fail

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After getting an intuition with the calculations we start to explore proofs

We will show

- addition is well defined

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- addition is well defined
- multiplication is well defined
- glimpses into proofs of Fermat's little Theorem and Euler's Theorem

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- Our favorite High School problems

$$2^{2018} \equiv 4 \pmod{7}$$

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- The beauty of mathematics (of course)
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$$2^{2018} \equiv 4 \pmod{7}$$

- The Hill cipher
- RSA (need another class for this one)
- Factorization of polynomials (again another class needed)

Thanks to

- The College at Old Westbury
- Institute for Creative Problem Solving
institutecreativeproblemsolving.org
- MERIT instituteofmerit.com
- the students
- the organizers of this session