Math 3160 - Test 1 Review

1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

(a) $$\begin{cases} x_1 & -2x_2 & & & -6x_5 & = 0 \\ & x_2 & +x_3 & +6x_4 & & = 5 \\ & 2x_2 & & +6x_4 & +x_5 & = 4 \\ & x_2 & -x_3 & & +x_5 & = -1 \end{cases}$$

(b) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & & x_3 & = 0 \\ x_1 & +x_2 & +2x_3 & = 0 \end{cases}$$

(c) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ -x_1 & -x_2 & +3x_3 & = 2 \\ x_1 & -3x_2 & +7x_3 & = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

(a) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$$

(b) $$\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$$

2 Matrices, Determinants and Cramer's Rule

3. Let $A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the determinant of the matices A, B, C and D

(b) Compute D^3 and D^{-1}.

(c) Compute C^TC. What kind of matrix is C^TC?

4. Solve the following equations for X assuming any matrix has an inverse. Let A, B, C, X be $n \times n$ matrices and let $\mathbf{u}$ be an $n \times 1$ vector.

   (a) $AX = BX - A$

   (b) $AX = 2X - A$

   (c) $A\mathbf{u} = 2\mathbf{u} + B$

5. Solve the following using Cramer's rule.

   (a) $\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$

   (b) $\begin{cases} 2x & -2y & = 2 \\ -x & -3y & = 2 \end{cases}$

3 Vectors

6. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$. and let $P(1, 1, 1)$ and $Q(0, -4, 0)$ be two points in $\mathbb{R}^3$.

   (a) Find a vector that is parallel to $\mathbf{v}$ and unit.

   (b) Compute $\|2\mathbf{v} - \mathbf{w}\|$.

   (c) Compute the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (d) Find the equation of a plane containing P and with normal vector $\mathbf{v}$.

7. Find parametric equations for the line (in $\mathbb{R}^3$) so that

   (a) the line contains the point $P(0, 1, 2)$ and $Q(1, 2, 3)$.

   (b) the line contains the point $P(-5, 1, 3)$ and is parallel to the vector $(1, 2, 3)$.

8. Find equation for the plane (in $\mathbb{R}^3$) so that

   (a) the plane contains the point $P(2, 2, 2)$, $Q(1, 2, 3)$ and $R(1, -1, 0)$.

   (b) the plane contains the origin and is perpendicular to the vector $(1, 2, 3)$.

9. Define the planes P_1 and P_2 as follows:

$$P_1 : x - 2y + z = 12$$

$$P_2 : 3x - 3y + z = 4$$

   (a) What are the two normal vectors for the above planes.
   (b) Find the angle between the two above planes.
   (c) Find two points one on each of the above planes.
   (d) Find set of all points that lay in both planes.

10. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$.

   (a) Compute $\mathbf{v} \times \mathbf{w}$.
   (b) Compute $\mathbf{w} \times \mathbf{v}$.
   (c) Compute $\mathbf{w} \times \hat{\imath}$.
   (d) Compute $(\sin(\theta), \cos(\theta), 1) \times (\cos(\theta), -\sin(\theta), 0)$.
   (e) What is the area contained within the parallelogram formed by the vectors $\mathbf{v}$ and $\mathbf{w}$.
   (f) What is the area contained within the triangle defined by the three vertices $P(1, 0, 1)$, $P(-2, 0, 0)$ and the origin?
   (g) what is the volume of the parallelpiped formed by the three vectors $\mathbf{v}$, $\mathbf{i}$ and $\mathbf{w}$.

11. Solve the following systems of linear equation using $A\mathbf{x} = \mathbf{b}$ and the inverse of A.

   (a) $\begin{cases} x_1 & -2x_2 & = 0 \\ x_1 & +x_2 & = 5 \end{cases}$

   (b) $\begin{cases} x_1 & -2x_2 & & = 0 \\ & x_2 & +x_3 & = 5 \\ & 2x_2 & & = 4 \end{cases}$

4 Couple of new problems

12. Compute the following Let $\mathbf{u} = (-2/3, 1/3, 2/3)$, $\mathbf{v} = (1, 2, 3)$ and let $\mathbf{w} = (1, 2, 3)$.

   (a) $Proj_{\mathbf{w}}\mathbf{v}$

(b) The component of $\mathbf{v}$ parallel to $\mathbf{w}$, and the component of $\mathbf{v}$ orthogonal to $\mathbf{w}$. We called these $\mathbf{C_1}$ and $\mathbf{C_2}$ in class.

(c) The component of $\mathbf{v}$ parallel to $\mathbf{u}$, and the component of $\mathbf{v}$ orthogonal to $\mathbf{u}$.