Answers For Test 2 Review

23.

$$A = \begin{bmatrix} -1 & 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & 2 & 1 \end{bmatrix}$$

DON'T WORRY ABOUT THE ROW SPACE.

(a) COL (A).

Row reduce $\begin{bmatrix} -1 & 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & 2 & 1 \end{bmatrix} \xrightarrow[\text{skipped}]{\text{steps}} \cdots \longrightarrow \begin{bmatrix} 1 & 0 & 2/5 & -1 & 2/5 \\ 0 & 1 & 1/5 & 1 & 1/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\text{COL}(A) = \text{SPAN}\left(\left\{\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\right\}\right)$$

(b) $\text{DIM}(\text{COL}(A)) = 2$

(c) $\left[\begin{array}{ccccc|c} -1 & 2 & 0 & 3 & 0 & 0 \\ 2 & 1 & 1 & -1 & 1 & 0 \\ 1 & 3 & 1 & 2 & 1 & 0 \end{array}\right] \xrightarrow{\text{Steps skipped}} \begin{array}{cccccc} & & x_3=t & x_4=s & x_5=u & \\ \end{array}\left[\begin{array}{ccccc|c} 1 & 0 & 2/5 & -1 & 2/5 & 0 \\ 0 & 1 & 1/5 & 1 & 1/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

FREE

$$x_2 + \tfrac{1}{5}x_3 + 1x_4 + \tfrac{1}{5}x_5 = 0$$

$$x_2 = -\tfrac{1}{5}t - s - \tfrac{1}{5}u$$

$$x_1 + \tfrac{2}{5}x_3 - 1x_4 + \tfrac{2}{5}x_5 = 0$$

$$x_1 = -\tfrac{2}{5}t + s - \tfrac{2}{5}u = 0$$

So $\text{NULL}(A) = S = \left\{\begin{bmatrix} -\frac{2}{5}t + s - \frac{2}{5}u \\ -\frac{1}{5}t - s - \frac{1}{5}u \\ t \\ s \\ u \end{bmatrix} : s,t,u \in \mathbb{R}\right\}$

$$= \left\{\begin{bmatrix} -2/5\,t \\ -1/5\,t \\ t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} s \\ -s \\ 0 \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -2/5\,u \\ -1/5\,u \\ 0 \\ 0 \\ u \end{bmatrix} : s,t,u \in \mathbb{R}\right\} = \left\{t\begin{bmatrix} -2/5 \\ -1/5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -2/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \end{bmatrix} : s,t,u \text{ etc}\right\}$$

$$= \text{SPAN}\left\{\begin{bmatrix} -2/5 \\ -1/5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$$

(d) $\text{DIM}(\text{NULL}(A)) = 3$

24. $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y+z \\ x-z \end{bmatrix}$

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

(b) COL(A) = RANGE(T)

R.R

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \to \cdots \to \begin{bmatrix} \boxed{1} & 0 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$= \text{SPAN}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

BASIS = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

(c) FOR the NULL SPACE R.R. $\left[\begin{array}{ccc:c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right] \to \cdots \to \left[\begin{array}{ccc:c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

FREE

$z = t$

$x - z = 0$
$x - t = 0$
$x = t$

$y + z = 0$
$y = -z$
$y = -t$

$S = \left\{ \begin{bmatrix} t \\ -t \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \left\{ t\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$

BASIS = $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

(d) DIM (COL (A)) = 2 , DIM (NULL (A)) = 1

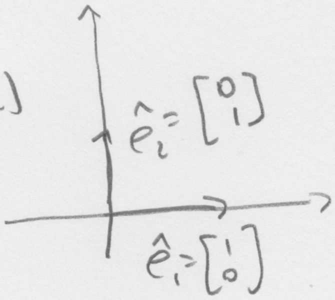
(e) DOMAIN (T) = $\mathbb{R}^3$ CODOMAIN (T) = $\mathbb{R}^3$

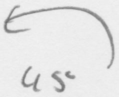
DIM (DOMAIN (T)) = 3 DIM (CODOMAIN (T)) = 3

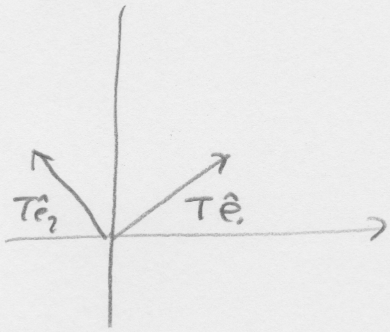
Don't worry about RANK or NULLITY.

25. $T: \mathbb{R}^2 \to \mathbb{R}^2$

(a)





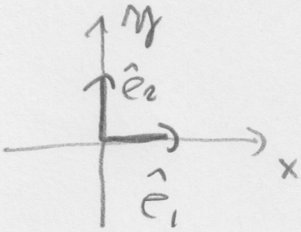


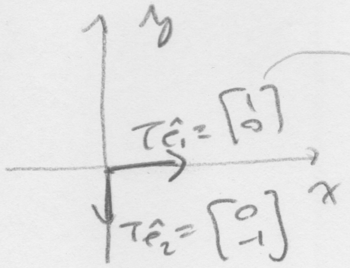
$$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30 & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

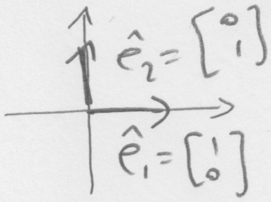
(b) T

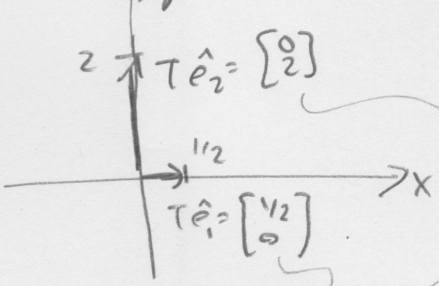




$$A = \begin{bmatrix} T\hat{e}_1 & T\hat{e}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

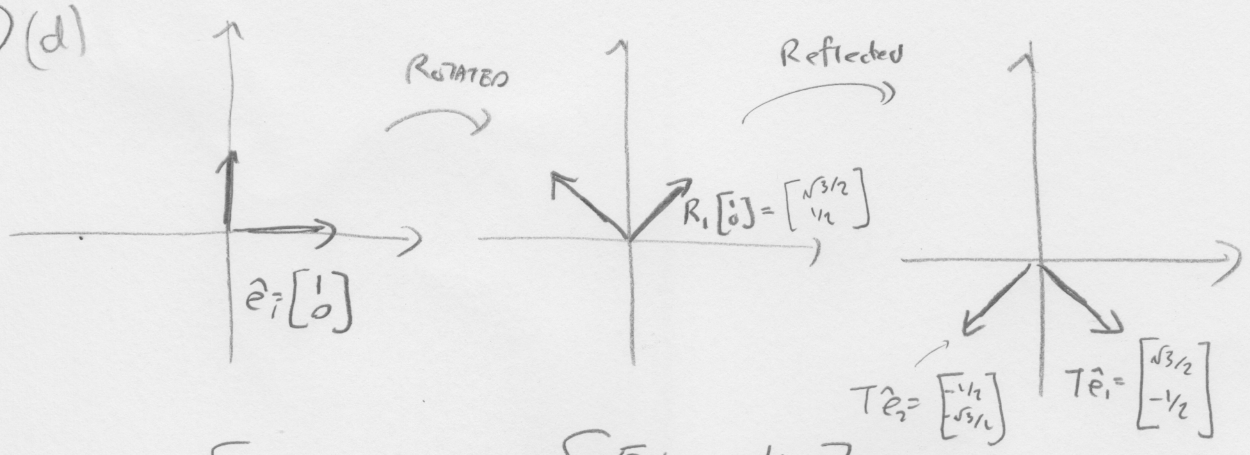
(c) T





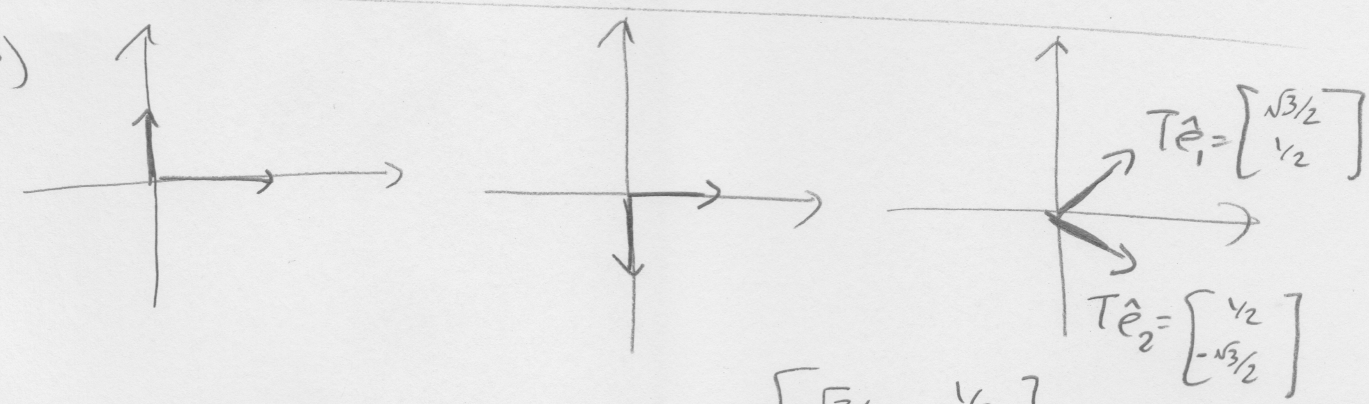
$$A = \begin{bmatrix} T\hat{e}_1 & T\hat{e}_2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

25. (d)



$$T = \begin{bmatrix} T\hat{e}_1 & Te_2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

(e)



$$T = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix}.$$

(f) $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$A = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$