Math 3160 - Test 1 Review

1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

(a) $$\begin{cases} x_1 - 2x_2 - 6x_5 = 0 \\ x_2 + x_3 + 6x_4 = 5 \\ 2x_2 + 6x_4 + x_5 = 4 \\ x_2 - x_3 + x_5 = -1 \end{cases}$$

(b) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$$

(c) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ -x_1 - x_2 + 3x_3 = 2 \\ x_1 - 3x_2 + 7x_3 = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

(a) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ -x_2 + 3x_3 = 2 \\ -3x_2 + 7x_3 = 2 \end{cases}$$

(b) $$\begin{cases} 2x_1 - 2y = 2 \\ -x_1 - 3y = 2 \end{cases}$$

2 Matrices, Determinants and Cramer's Rule

3. Let $A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the determinant of the matices A, B, C and D

(b) Compute D^3 and D^{-1}.

(c) Compute C^TC. What kind of matrix is C^TC?

4. Solve the following equations for X assuming any matrix has an inverse. Let A, B, C, X be $n \times n$ matrices and let $\mathbf{u}$ be an $n \times 1$ vector.

   (a) $AX = BX - A$

   (b) $AX = 2X - A$

   (c) $A\mathbf{u} = 2\mathbf{u} + B$

5. Solve the following using Cramer's rule.

   (a) $\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$

   (b) $\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$

3 Vectors

6. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$. and let $P(1, 1, 1)$ and $Q(0, -4, 0)$ be two points in $\mathbb{R}^3$.

   (a) Find a vector that is parallel to $\mathbf{v}$ and unit.

   (b) Compute $\|2\mathbf{v} - \mathbf{w}\|$.

   (c) Compute the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (d) ~~Find the equation of a plane containing P and with normal vector $\mathbf{v}$.~~

7. ~~Find parametric equations for the line (in $\mathbb{R}^3$) so that~~

   (a) ~~the line contains the point $P(0, 1, 2)$ and $Q(1, 2, 3)$.~~

   (b) ~~the line contains the point $P(-5, 1, 3)$ and is parallel to the vector $(1, 2, 3)$.~~

8. ~~Find equation for the plane (in $\mathbb{R}^3$) so that~~

   (a) ~~the plane contains the point $P(2, 2, 2)$, $Q(1, 2, 3)$ and $R(1, -1, 0)$.~~

   (b) ~~the plane contains the origin and is perpendicular to the vector $(1, 2, 3)$.~~

9. ~~Define the planes P_1 and P_2 as follows:~~

$$P_1 : x - 2y + z = 12$$

$$P_2 : 3x - 3y + z = 4$$

   (a) ~~What are the two normal vectors for the above planes.~~

   (b) ~~Find the angle between the two above planes.~~

   (c) ~~Find two pointsone on each of the above planes.~~

   (d) ~~Find set of all points that lay in both planes~~.

10. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$.

   (a) Compute $\mathbf{v} \times \mathbf{w}$.

   (b) Compute $\mathbf{w} \times \mathbf{v}$.

   (c) Compute $\mathbf{w} \times \hat{\imath}$.

   (d) Compute $(\sin(\theta), \cos(\theta), 1) \times (\cos(\theta), -\sin(\theta), 0)$.

   (e) What is the area contained within the parallelogram formed by the vectors $\mathbf{v}$ and $\mathbf{w}$.

   (f) What is the area contained within the triangle defined by the three vertices $P(1, 0, 1)$, $P(-2, 0, 0)$ and the origin?

   (g) what is the volume of the parallelpiped formed by the three vectors $\mathbf{v}$, $\mathbf{i}$ and $\mathbf{w}$.

11. Solve the following systems of linear equation using $A\mathbf{x} = \mathbf{b}$ and the inverse of A.

   (a) $\begin{cases} x_1 & -2x_2 & = 0 \\ x_1 & x_2 & = 5 \end{cases}$

   (b) $\begin{cases} x_1 & -2x_2 & & = 0 \\ & x_2 & +x_3 & = 5 \\ & 2x_2 & & = 4 \end{cases}$