Hint. Five problems guaranteed to be on the final exam, one of each of technique of integration (u-sub, byparts, trig integral, trig sub and partial fractions).

## 1 Definition of the integral

1. Write the definition of the integral.
2. Using the definition of the integral compute

$$\int_1^4 3x^2\,dx$$

## 2 Applications of the integral

Some good functions to be able to graph here area lines, parabolas, $y = e^x$, $y = \ln(x)$, the trig functions and their translations (such as $y = -2x^2 + 1$). Later you should know the conic sections.

3. Let $a(t) = t^2, v(0) = 0$ and $s(0) = 16$.
   (a) Find $v(t)$ and $s(t)$.
   (b) When is the particle at rest?
   (c) When is the particle at ground height?
4. Find the area between $y = e^x$, $y = \frac{1}{2}x$, $y = 1$ and $y = e$.
5. Revolve the region about the $x$-axis and compute its volume. The region is inside of $y = x^2$, and $y = 2x$.
6. Revolve the region about the $x$-axis and compute its volume. The region is inside of $x = y^2$, and $x = 9$.
7. Revolve the region about the $x$-axis and compute its volume. The region is inside of $x = y^2$, and $y = x - 2$.

## 3 Techniques of integration

8. $\int xe^{x^2}\,dx$

9. $\int x\sqrt{x^2+1}\,dx$

10. $\int \frac{x}{x^2+1}\,dx$

11. $\int \frac{1}{x^2+1}\,dx$

12. $\int [tan(3x)+1]^{1/3}\sec^2(3x)\,dx$

13. $\int \frac{\sqrt{\ln(x)+1}}{x}\,dx$

14. $\int \frac{\sqrt{\ln(x)+1}}{x}\,dx$

15. $\int xe^{2x}\,dx$

16. $\int x\sin(3x)\,dx$

17. $\int \arctan(x)\,dx$ Hint: $u = \arctan(x)$ and $dv = 1\,dx$

18. $\int x^2e^x\,dx$

19. $\int x^2\ln(x)\,dx$

20. $\int x\ln(x+1)\,dx$

21. $\int \sin^2(x)\,dx$

22. $\int \sin^3(x)\,dx$

23. $\int \cos^2(x)\,dx$

24. $\int \cos^2(x)\sin(x)\,dx$

25. $\displaystyle\int \frac{1}{\sqrt{4x^2-9}}\,dx$ Hint. No triangle needed.

26. $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx$

27. $\displaystyle\int \frac{1}{(x^2+25)^{3/2}}\,dx$

28. $\displaystyle\int \frac{1}{x^2-4}\,dx$

29. $\displaystyle\int \frac{1}{x^3-x^2}\,dx$

30. $\displaystyle\int \frac{1}{x^4-x^2}\,dx$

31. $\displaystyle\int \frac{2x^2+3x+3}{(x+2)(x^2+1)}\,dx$

## 4 Sequences and Series

32. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$

33. $\displaystyle\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$

34. $\displaystyle\sum_{k=1}^{\infty} \sqrt{k+1} - 2\sqrt{k} + \sqrt{k-1}$

35. $\displaystyle\sum_{k=-2}^{\infty} \frac{1}{3^n}$

36. $\displaystyle\sum_{k=-2}^{\infty} \frac{1}{3^{-n}}$

37. $\displaystyle\sum_{k=0}^{\infty} \frac{7^n}{3^n}$

38. $\sum_{k=1}^{\infty} \frac{k+2}{2k+3}$

39. $\sum_{k=1}^{\infty} \left(1 + \frac{1}{2k+3}\right)^k$

40. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use integral test

41. $\sum_{k=7}^{\infty} \frac{1}{k\ln(k)}$ Use integral test

42. $\sum_{k=1}^{\infty} \frac{1}{k^2}$ Use p-Series test

43. $\sum_{k=1}^{\infty} \frac{1}{k}$ Use p-Series test

44. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ Use p-Series test

45. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use LCT

46. $\sum_{k=1}^{\infty} \frac{3k}{k^3+1}$

47. $\sum_{k=1}^{\infty} \frac{\sqrt{k^2+5}}{k^2+1}$

48. $\sum_{k=1}^{\infty} (-1)^n \frac{k}{k^2+1}$

49. $\sum_{k=1}^{\infty} (-1)^n \left(1 - \frac{1}{k}\right)^k$

50. $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ Use Ratio test

51. $\sum_{k=1}^{\infty} \frac{2^k}{e^k + 1}$

52. $\sum_{k=1}^{\infty} \frac{k^2}{e^k}$

53. $\sum_{k=1}^{\infty} \frac{1}{k^k}$

54. $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

55. $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

56. $\sum_{k=1}^{\infty} (\frac{k}{2k+1})^k$

57. $\sum_{k=1}^{\infty} (\frac{3k^2+1}{2k^2+17k+91})^k$

58. $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^{k^2}$

## 5 Power and Taylor Series

59. What are the radii and intervals of convergence of the following power series.

    (a) $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

    (b) $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

    (c) $\sum_{n=1}^{\infty} \frac{1}{n^2} (x-5)^n$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} x^n$

(e) $\displaystyle\sum_{n=0}^{\infty} x^n$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

60. Find the Taylor series from the definition

    (a) $f(x) = \sin(2x)$ at $x = 0$

    (b) $f(x) = e^x$ at $x = 1$

    (c) $f(x) = \frac{1}{1-x}$ at $x = 0$

    (d) $f(x) = \ln(x)$ at $x = 1$

61. Find the Taylor series from a known Taylor series. Using only the taylor series for $e^x$, $\sin(x)$, $\cos(x)$, and $\frac{1}{1-x}$

    (a) $f(x) = x\sin(x^2) - x^3$

    (b) $f(x) = \frac{\sin(x)}{x}$

    (c) $f(x) = \frac{e^{x^2} - 1 - x^2}{x^4}$

## 6 Parametric Equations

62. Find the parametric equation for the following equations given in rectangular coordinates.

    (a) $y = x^2$

    (b) $y = 3x - 1$

    (c) $y^2 + y = 2x + 2$

63. Find equation in rectangular coordinates for the following equations given in the parametrically.

    (a) $x = 3t$ and $y = 2t - 1$

    (b) $x = 3t$ and $y = 2t^2 - 1$

    (c) $x = \cos(t)$ and $y = 3\sin(t)$

64. Graph the following parametric equations

    (a) $x = 3t$ and $y = 2t - 1$
    (b) $x = 3t$ and $y = 2t^2 - 1$
    (c) $x = \cos(t)$ and $y = 3\sin(t)$
    (d) $x = t\cos(t)$ and $y = t\sin(t)$

65. For the following find the eqaution of the tangent line at the given point.

    (a) $x = 3t$ and $y = 2t - 1$ at $P = (6, 3)$
    (b) $x = 3t$ and $y = 2t^2 - 1$ at $t = 1$
    (c) $x = \cos(t)$ and $y = 3\sin(t)$ at $t = \pi/4$
    (d) $x = t\cos(\pi t)$ and $y = t\sin(\pi t)$ at $t = 1$

## 7 Polar Coordinates

66. Graph the following polar equations.

    (a) $r = 3$
    (b) $r = 4\sin(\theta)$
    (c) $r = \sin(2\theta)$
    (d) $r = 1 + 2\sin(\theta)$

67. For the folowing find the equation of the tangent line at the given point.

    (a) $r = 3$ at $P = (x_0, y_0) = (\frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}})$
    (b) $r = 4\sin(\theta)$ at $\theta = \pi/4$
    (c) $r = \sin(2\theta)$ at $\theta = \pi/2$
    (d) $r = 1 + 2\sin(\theta)$ at $\theta = \pi/3$

## 8 Conic Sections

68. Graph the given conic sections.

    (a) $x^2 + \frac{y^2}{4} = 1$

(b) $x^2 - \frac{y^2}{4} = 1$

(c) $x + \frac{y^2}{4} = 1$

(d) $x^2 + 4y^2 = 1$

(e) $(x-1)^2 - \frac{y^2}{4} = 1$