Math 6250 Homework 3

Name:____

1. Show $(0,1) \sim (0,\infty)$. And show $(0,1] \sim [0,2)$. That is find a 1-1 and onto function from $(0,1)$ to $(0,\infty)$. Prove your function is 1-1 and onto.

2. Let A, B be nonempty bounded subsets of $\mathbb{R}$. Define $A+B = \{a+b | a \in A, b \in B\}$ $-A = \{-a | a \in A\}$

   (a) For $A = (1,3)$ and $B = [-4,-1]$. Compute $A+B$ and $-A$.

   (b) For $A = (1,3)$ and $B = [-4,-1]$. Compute $\sup(A)$, $\sup(B)$, $\sup(A+B)$ and $\sup(-A)$.

   (c) Prove the following fact.
   For any A, B nonempty bounded subsets of $\mathbb{R}$ we have that
   $$\sup(A) + \sup(B) = \sup(A+B).$$

   (d) Guess a similar fact about the $\sup(-A)$.

3. Prove the triangle inequality. That is for all $x, y \in \mathbb{R}$
   $$|x+y| \leq |x| + |y|.$$
   Hint: It is easier to show $|x+y|^2 \leq (|x|+|y|)^2$ by looking at various cases.

4. How have we defined the reals? The reals are also the only complete ordered field. What are the definitions for 1. complete. 2. ordered and 3. field. Look these up.

5. Prove the sup of set is unique.

6. The Completeness Axiom is a statemnt about the sup of a set in $\mathbb{R}$. State an anlagous result for infs in $\mathbb{R}$. Prove it.

7. Calculate the fourth roots of i. And Calculate the cube roots of $-1+i$

8. Use Euler's to show: $\cos(2\theta) = \cos^2\theta - \sin^2\theta$.