Math 6250 Homework 2

Name:_____

1. Prove for $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \frac{1}{x^2 + 1}$$

   is neither injective nor surjective.

2. Prove for $f : [0, \infty) \to (0, 1)$ given by

$$f(x) = \frac{1}{x^2 + 1}$$

   is injective and surjective.

3. Prove for $f : A \to B$ and $g : B \to C$ that

   (a) If f and g are injective then $g \circ f$ is injective.

   (b) If f and g are surjective then $g \circ f$ is surjective.

4. Write the definition of countable and uncountable, infinite and finite. Where do these overlap?

5. Show $A \sim \mathbb{N}$ where

$$A = \{1, 2, 3\} \times \mathbb{N}.$$

6. Show $\mathbb{Q} \sim \mathbb{N}$. You may wish to watch a video or read your book.

7. Show $\mathbb{R}$ is not countable! You may wish to watch a video or read your book. Now that you have competed Problem 6 and Problem 7 update Problem 4