Math 6250 Homework 1

Name:____

1. For the following series

$$S(n) = \sum_{k=1}^{n} 3k^2 - 3k + 1$$

   (a) Compute $S(1), S(2), S(3)$, and $S(4)$

   (b) Conjecture a formula for $S(n)$ for all $n \in \mathbb{N}$.

   (c) Prove using induction the formula you conjectures above is correct.

2. State (as we did in class) and Prove Pascal's Lemma. Here you do not need induction.

3. Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by

$$a\mathcal{R}b \iff 5|(a-b).$$

   (a) Note $3\mathcal{R}28$ since it is true that $5|(3-28)$ and 3 does not relate to 27 since it is false that $5|(3-27)$. Find three other pairs of integers that relate to each other.

   (b) State $\mathcal{R}$ is reflexive and prove it. That is, say

   For all $z \in \mathbb{Z}$ we have $z\mathcal{R}z$.

   Proof. $\cdots$ □

   (c) State $\mathcal{R}$ is symmetric and prove it.

   (d) State $\mathcal{R}$ is transitive and prove it.

4. Let $\mathcal{R}$ be the relation on the set $\mathbb{N} \times \mathbb{N}$ defined by

$$(a_1, b_1)\mathcal{R}(a_2, b_2) \iff a_1 + b_2 = a_2 + b_1$$

   (a) Note the ordered pair $(1,2)\mathcal{R}(8,9)$ since $1+9 = 2+8$. Find four other ordered pairs of natural numbers that relate to each other.

(b) State $\mathcal{R}$ is reflexive and prove it. That is, say

For all pairs $(a, b) \in \mathbb{N} \times \mathbb{N}$ we have $(a, b)\mathcal{R}(a, b)$.

Proof. $\cdots$ $\square$

(c) State $\mathcal{R}$ is symmetric and prove it.

(d) State $\mathcal{R}$ is transitive and prove it.

5. Let $\mathcal{R}$ be the relation on the set $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ defined by

$$(a_1, b_1)\mathcal{R}(a_2, b_2) \iff a_1 b_2 = a_2 b_1$$

(a) Note the ordered pair $(6, 4)\mathcal{R}(9, 6)$ since $6 \cdot 6 = 4 \cdot 9$. Find four other ordered pairs of integers that relate to each other.

(b) State $\mathcal{R}$ is reflexive and prove it. That is, say

For all pairs $(a, b) \in \mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ we have $(a, b)\mathcal{R}(a, b)$.

Proof. $\cdots$ $\square$

(c) State $\mathcal{R}$ is symmetric and prove it.

(d) State $\mathcal{R}$ is transitive and prove it.

6. This problem refers to Problem 3. Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by

$$a\mathcal{R}b \iff 5|(a - b).$$

Is the usual operation of addition **well defined**? State what it means for addition to be well defined over this relation and prove that addition is well defined.