1 Integrals

1. $\displaystyle\iint_R x + y\, dA$ over the region defined by $x + y = 2$ and the coordinate axes.

2. $\displaystyle\iint_R xy\, dA$ over the region defined by $y = x^2$ and the line $y = x + 1$.

3. $\displaystyle\iint_R e^{x^2}\, dA$ over the region defined by $y = -x$, $y = 2x$ and the vertical line $x = 4$.

4. $\displaystyle\iint_R e^{x^2+y^2}\, dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ in the third quadrant.

5. $\displaystyle\iint_R \sqrt{\frac{\tan^{-1}(y/x)}{x^2 + y^2}}\, dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ above the lines $y = -x$ and $y = x$.

6. Find the volume below the paraboloid $z = 12 - x^2 - y^2$ and above the xy-plane.

7. $\displaystyle\iint_R \sin(x - y)\cos(x + y)\, dA$ over the region defined the lines $y = x + 2$, $y = x + 4$, $y = -x$ and $y = -x + 3$. Hint the change of variables is $u = x - y$ and $v = x + y$.

8. $\displaystyle\iint_R \frac{x - y}{2x + y}\, dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -2x + 2$ and $y = -2x + 3$.

9. $\displaystyle\iint_R xy\, dA$ over the region defined the graphs of $xy = 1$, $xy = 3$ and the lines $y = x$ and $y = 3x$ (first quadrant). Hint $x = u/v$ and $y = v$.

10. $\displaystyle\iint_R (x - y)e^{x^2-y^2}\, dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -x$ and $y = -x + 3$.

11. $\displaystyle\iint_R e^{x^2+4y^2}\, dA$ over the region defined by the portion of the ellipse $\frac{x^2}{4} + y^2 = 1$ in the third quadrant. Hint use the change of variables $x = 2v\cos(u)$ and $y = v\sin(u)$. And note I had $\pi \leq u \leq \frac{3\pi}{2}$

2 Line Integrals

12. $\displaystyle\int_C x\, dx$. Let C be line segment from $(0, 1)$ to $(3, 2)$.

13. $\int_C xy\, ds$. Let C be line segment from $(0,1)$ to $(3,2)$.

14. $\int_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be line segment from $(0,1)$ to $(3,2)$.

15. $\int_C x\, dy$. Let C be line segment from $(0,1)$ to $(3,2)$.

16. $\oint_C xy\, dx$. Let C be the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

17. $\oint_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

18. $\oint_C \langle 1, xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2 + y^2 = 4$ traced counter-clockwise.

19. $\oint_C -x + yds$. Let C be the circle $x^2 + y^2 = 4$ traced counter-clockwise.

3 Conservative Fields and the FTVC

20. Graph the following fields.
    (a) $\mathbf{F}(x,y) = \langle 2x - y, x\rangle$
    (b) $\mathbf{F}(x,y) = \langle y, x\rangle$
    (c) $\mathbf{F}(x,y) = \langle 2 - y, 1\rangle$

21. is the field conservative? If yes find it's potential.
    (a) $\mathbf{F}(x,y) = \langle x, y\rangle$
    (b) $\mathbf{F}(x,y) = \langle y, x\rangle$
    (c) $\mathbf{F}(x,y) = \langle y\cos(xy), x\cos(xy)\rangle$
    (d) $\mathbf{F}(x,y) = \langle 2 - y, 1\rangle$

22. Use the FTVC to solve
    (a) $\int_C \langle y^2 + 6x, 2xy\rangle \cdot d\mathbf{r}$. Let C be the line segment from $(-2,0)$ to $(2,0)$.
    (b) $\int_C \langle y^2 + 6x, 2xy\rangle \cdot d\mathbf{r}$. Let C trace the parabola $y = 4 - x^2$ from $(-2,0)$ to $(2,0)$.
    (c) $\int_C \langle xe^{x^2}, 1\rangle \cdot d\mathbf{r}$. Let C be the line segment from $(0,0)$ to $(1,1)$.

(d) $\oint_C \langle 2xe^{x^2+y^2}, 2ye^{x^2+y^2}\rangle \cdot d\mathbf{r}$. Let C trace the circle $x^2+y^2=4$ from $(-2,0)$ to $(2,0)$.

4 Green's Theorem

23. $\oint_C \langle x, -y\rangle \cdot d\mathbf{r}$. Let C be outside of the rectangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ to $(1,0)$ and then back to $(0,0)$.

24. $\oint_C \langle e^{x^3} - xy, e^{y^3} - y\rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

25. $\oint_C \langle \cos(x^2)+y, \cos(y^2)+xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2+y^2=4$ traced counter-clockwise.

5 Div/Grad/Curl

26. Define
$$f(x,y,z) = x^3 - yz^2 \text{ and } \mathbf{F}(x,y,z) = \langle x^3, yz^2, xy\rangle.$$
Compute the following, if possible, and if not possible state why.

   (a) $\text{div}(f(x,y,z))$
   (b) $\text{grad}(f(x,y,z))$
   (c) $\text{curl}(f(x,y,z))$
   (d) $\text{div}(\mathbf{F}(x,y,z))$
   (e) $\text{grad}(\mathbf{F}(x,y,z))$
   (f) $\text{curl}(\mathbf{F}(x,y,z))$
   (g) $\nabla \cdot \mathbf{F}(x,y,z)$
   (h) $\nabla \times (\nabla \cdot \mathbf{F}(x,y,z))$
   (i) $\nabla \times (\nabla f(x,y,z))$

6 Surface Integrals

27. For the following exercises, let S be the hemisphere $x^2+y^2+z^2=4$, with $z \geq 0$, and evaluate each surface integral, in the counterclockwise direction.

   (a) $\iint z\,dS$

(b) $\iint (x-2y)dS$

28. For the following exercises, evaluate

$$\iint_S \mathbf{F}\cdot\mathbf{N}dS = \iint_S \mathbf{F}\cdot d\mathbf{S}$$

for vector field $\mathbf{F}$, where $\mathbf{N}$ is an outward normal vector to surface S.

(a) $F(x,y,z) = xi + 2yj - 3zk$, and S is that part of plane $15x - 12y + 3z = 6$ that lies above unit square $0 \le x \le 1$, $0 \le y \le 1$.

(b) $F(x,y,z) = xi + yj$, and S is the hemisphere $z = \sqrt{1-x^2-y^2}$.

(c) $F(x,y,z) = x^2i + y^2j + z^2k$, and S is the portion of plane $z = y+1$ that lies inside cylinder $x^2+y^2=1$.

7 Stokes' Theorem

29. For the following compute both $\iint_S \operatorname{curl}((\mathbf{F}))\cdot d\mathbf{S}$ and $\int_{\partial S}\mathbf{F}\cdot d\mathbf{r}$. Verify they are the same.

(a) $F(x,y,z) = y^2i + z^2j + x^2k$; S is the first-octant portion of plane $x+y+z=1$.

(b) $F(x,y,z) = zi + xj + yk$; S is the hemisphere $z = (9-x^2-y^2)^{1/2}$.

(c) $F(x,y,z) = y^2i + 2xj + 5k$; S is the hemisphere $z = (9-x^2-y^2)^{1/2}$.

(d) $F(x,y,z) = zi + 2xj + 3yk$; S is upper hemisphere $z = \sqrt{9-x^2-y^2}$.

(e) $F(x,y,z) = (x+2z)i + (y-x)j + (z-y)k$; S is a triangular region with vertices $(3,0,0)$, $(0,3/2,0)$, and $(0,0,3)$.