1) $\vec{r}(t) = \langle t^2 - t,\ t^3 - 3t^2 + 3 \rangle$

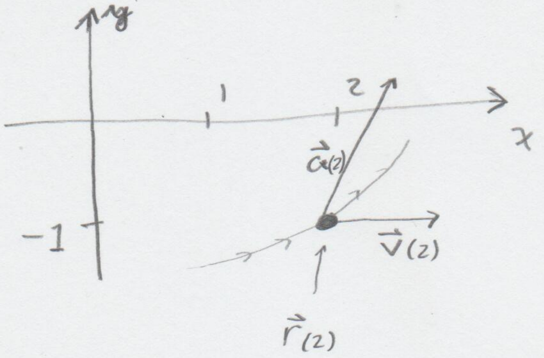
$\vec{v} = \vec{r}\,'(t) = \langle 2t,\ 3t^2 - 6t \rangle$

$\vec{a} = \vec{r}\,''(t) = \langle 2,\ 6t - 6 \rangle$

$\vec{r}(2) = \langle 2,\ 8 - 12 + 3 \rangle = \langle 2, -1 \rangle$

$\vec{r}\,'(2) = \vec{v}(2) = \langle 4, 0 \rangle$

$\vec{r}\,''(2) = \vec{a}(2) = \langle 2, 6 \rangle$



2) $\vec{r}(t) = \langle \sin(e^{-t}),\ \cos(e^{-t}) \rangle$

(a) SPEED

$$\frac{ds}{dt} = \| \vec{r}\,'(t) \| = \| \langle -e^{-t}\cos(e^{-t}),\ -e^{-t}\sin(e^{-t}) \rangle \|$$

$$= \| e^{-t} \langle \cos(e^{-t}),\ \sin(e^{-t}) \rangle \| = |e^{-t}| \cdot \sqrt{\cos^2(e^{-t}) + \sin^2(e^{-t})}$$

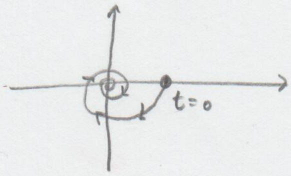
$$= e^{-t}$$

(b) $s = \int_0^1 \frac{ds}{dt}\,dt = \int_0^1 e^{-t}\,dt = -e^{-t}\Big|_0^1 = -e^{-1} + e^0 = 1 - \frac{1}{e}.$

(c) $s = \int_0^\infty \frac{ds}{dt}\,dt = \lim_{b\to\infty} \int_0^b e^{-t}\,dt = \lim_{b\to\infty} -e^{-t}\Big|_0^b = \lim_{b\to\infty}\left[-e^{-b} + e^0\right]$

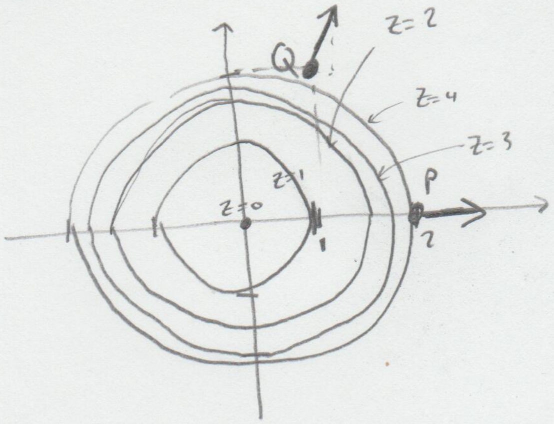
$$= \lim_{b\to\infty}\left[\frac{-1}{e^b} + 1\right] = 1.$$

(d)



(5.) $f(x,y) = x^2 + y^2$ $\quad P(0,2)$, $Q(1,2)$

(a) GRAPH CONTOURS at $z = -1, 0, 1, 2, 3, 4$



(b) compute

$$\vec{\nabla} f(x,y) = \langle 2x, 2y \rangle$$

(c)

$$\vec{\nabla} f(0,2) = \langle 0, 4 \rangle \qquad \|\vec{\nabla} f(0,2)\| = 4$$

$$\vec{\nabla} f(1,2) = \langle 2, 4 \rangle \qquad \|\vec{\nabla} f(1,2)\| = \sqrt{2^2+4^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

8. $f(x,y) = e^{xy^2+2} - xy^3 + 2$ | TANGENT PLANE at $P(-2,1)$

POINT $f(-2,1) = e^{-2(1)+2} - (-2)(1)^3 + 2 = 1 + 2 + 2 = 5$

$P(-2, 1, 5)$

NORMAL VECTOR $\vec{n} = \langle f_x(P), f_y(P), -1 \rangle$

$f_x(x,y) = y^2 e^{xy^2+2} - y^3$

$f_x(-2,1) = (1)^2 e^0 - (1)^3$

$= 0$

$f_y(x,y) = 2xy\, e^{xy^2+2} - 3xy^2$

$f_y(-2,1) = 2(-2)(1) e^0 - 3(-2)(1)^2$

$= -4 + 6 = 2$

$\vec{n} = \langle 0, 2, -1 \rangle$

TANGENT PLANE $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$0(x+2) + 2(y-1) + (-1)(z-5) = 0$

$z = 5 + 2(y-1)$

Approximate $f(-2.1, 0.8)$

the real function is about 5.0022...

true value

$z = 5 + 2(0.8 - 1)$

$= 5 - .4 = 4.6$

$= 4.6$

approximation

(9.) $f(x,y,z) = x^2y - xy^2 + z^3$

$P(1,2,3)$

POINT $(x_0, y_0, z_0, f(x_0,y_0,z_0))$

$f(1,2,3) = 1^2 \cdot 2 - 1 \cdot 2^2 + 3^3 = 2 - 4 + 27 = 25$

$P(1,2,3,25)$

NORMAL vector

$\vec{n} = \langle f_x(P), f_y(P), f_z(P), -1 \rangle$

$= \langle 0, -3, 27, -1 \rangle$

$f_x(x,y,z) = 2xy - y^2$ | $f_x(1,2,3) = 2(1)(2) - 4 = 0$

$f_y(x,y,z) = x^2 - 2xy$ | $f_y(1,2,3) = 1^2 - 2(1)(2) = -3$

$f_z(x,y,z) = 3z^2$ | $f_z(1,2,3) = 27$

$a(x - x_0) + b(y - y_0) + c(z - z_0)$

$+ d(w - w_0) = 0$

$$\boxed{0(x-1) + -3(y-2) + 27(z-3) + (-1)(w-25) = 0}$$

(12.) (a) $f(x,y) = x^2 - xy + y^3$

FIND CP

$f_x = 2x - y$ $\quad$ $f_y = -x + 3y^2$

$2x - y = 0$ $\quad$ $-x + 3y^2 = 0$

$y = 2x$ $\quad$ $-x + 3(2x)^2 = 0$

$-x + 12x^2 = 0$

$x(-1 + 12x) = 0$

$x = 0 \quad x = \frac{1}{12}$

$y = 0 \quad y = 1/6$

$CP_1 \ (0,0) \quad (1/12, 1/6) \leftarrow CP_2$

$f_{xx} = 2$

$f_{xy} = -1$

$f_{yy} = 6y$

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2 = 2(6y) - (-1)^2 = 12y - 1$

$\boxed{CP1}$ $(0,0)$

$D(0,0) = 12(0) - 1 = -1 < 0$ $\quad$ Saddle point

$\boxed{CP_2}$ $(1/12, 1/6)$

$D(1/12, 1/6) = 12(1/6) - 1 = 1$

$f_{xx}(\frac{1}{12}, \frac{1}{6}) = 2 > 0$

minimum.

(12 d) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2$ $\quad x+y+z-3w=4$

$\vec{\nabla} f = \lambda \vec{\nabla} g$ $\quad g(x,y,z,w) = x+y+z-3w-4$

$\langle 2x, 2y, 2z, 2w \rangle = \lambda \langle 1, 1, 1, -3 \rangle$ $\quad g = 0$

$2x = \lambda$ $\quad x = \lambda/2$

$2y = \lambda$ $\quad y = \lambda/2$

$2z = \lambda$ $\quad z = \lambda/2$

$2w = -3\lambda$ $\quad w = -3\lambda/2$

$x + y + z - 3w = 4$

$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2} - 3\frac{(-3\lambda)}{2} = 4$

$\frac{12\lambda}{2} = 4$

$\lambda = \frac{8}{12} = \frac{2}{3}$

$x = 1/3$

$y = 1/3$

$z = 1/3$

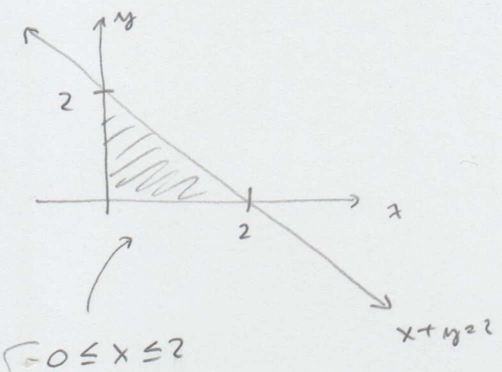
$w = -1$

at $(1/3, 1/3, 1/3, -1)$ f attains a minimum.

13. $\iint_R (x+y)\,dA$

R

$x+y=2$ & coordinate axes



$$\begin{cases} 0 \le x \le 2 \\ 0 \le y \le -x+2 \end{cases}$$

$$= \int_0^2 \left[\int_0^{-x+2} (x+y)\,dy \right] dx$$

$$= \int_0^2 \left[xy + \frac{1}{2}y^2 \right]_0^{-x+2} dx$$

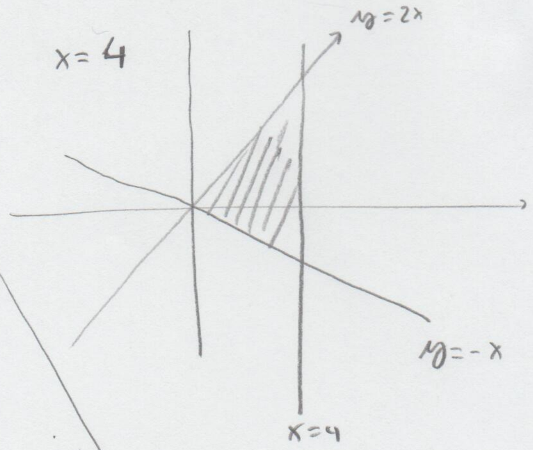
$$= \int_0^2 \left[x(-x+2) + \frac{1}{2}(-x+2)^2 - 0 \right] dx = \int_0^2 \left[-x^2 + 2x + \frac{1}{2}\left[x^2 - 4x + 4 \right] \right] dx$$

$$= \int_0^2 \left[-x^2 + \cancel{2x} + \frac{1}{2}x^2 - \cancel{2x} + 2 \right] dx = \int_0^2 \left(2 - \frac{1}{2}x^2 \right) dx = 2x - \frac{1}{6}x^3 \Big|_0^2$$

$$= \boxed{4 - \frac{8}{6} - 0}$$

15. $\iint_R e^{x^2}\,dA$

$y=-x$, $y=2x$, $x=4$



$0 \le x \le 4$

$-x \le y \le 2x$

$$= \int_0^4 \int_{-x}^{2x} e^{x^2}\,dy\,dx$$

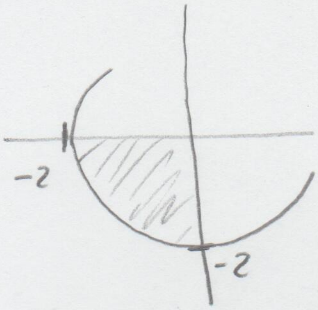
$$= \int_0^4 y e^{x^2} \Big|_{-x}^{2x} dx = \int_0^4 \left[2x - (-x) \right] e^{x^2} dx$$

$$= \int_0^4 3x e^{x^2} dx = \frac{3}{2} e^{x^2} \Big|_0^4 = \boxed{\frac{3}{2}\left[e^{16} - e^{0} \right]}$$

u-sub

$u = x^2$, $du = 2x\,dx$

(16.) $\iint e^{x^2+y^2}\, dA$ $\qquad x^2+y^2=4$ in third quadrant



$0 \le r \le 2$

$\pi \le \theta \le \frac{3\pi}{2}$

$$= \int_0^{3\pi/2}\int_0^2 \underbrace{e^{r^2}\, r\,dr}\, d\theta$$

u-sub, $u = r^2$, $du = 2r\,dr$

$$= \int_0^{3\pi/2} \frac{1}{2} e^{r^2}\Big|_0^2 \, d\theta = \frac{1}{2}\int_0^{3\pi/2} (e^4 - e^0)\, d\theta$$

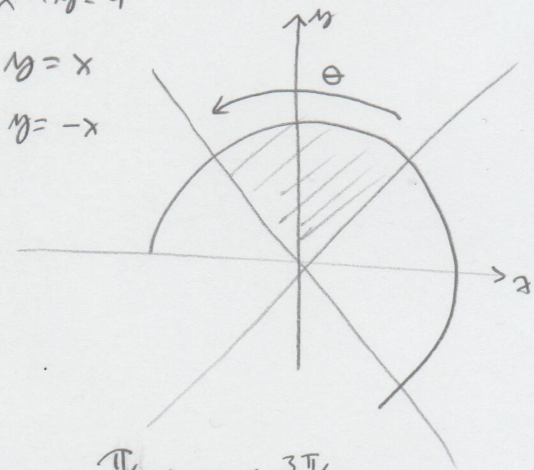
$$= \frac{1}{2}(e^4 - 1)\,\theta\Big|_0^{3\pi/2} = \frac{3\pi}{4}(e^4 - 1)$$

(17.) $\iint \sqrt{\dfrac{\tan^{-1}(y/x)}{x^2+y^2}}\, dA$

$x^2+y^2=4$

$y = x$

$y = -x$



$\pi/4 \le \theta \le 3\pi/4$

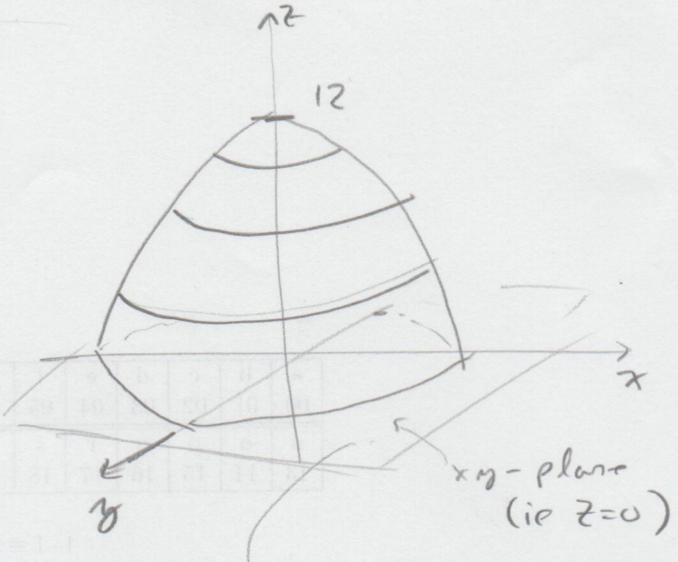
$0 \le r \le 2$

$$= \int_{\pi/4}^{3\pi/4}\int_0^2 \sqrt{\frac{\theta}{r^2}}\; r\,dr\, d\theta$$

$$= \int_{\pi/4}^{3\pi/4}\int_0^2 \theta^{1/2}\, dr\, d\theta = \int_{\pi/4}^{3\pi/4} r\,\theta^{1/2}\Big|_0^2 d\theta$$

$$= \int (2-0)\,\theta^{1/2}\, d\theta = 2\cdot\frac{2\theta^{3/2}}{3}\Big|_{\pi/4}^{3\pi/4} = \frac{4}{3}\left(\left(\frac{3\pi}{4}\right)^{3/2} - \left(\pi/4\right)^{3/2}\right)$$

18. $z = 12 - x^2 - y^2$



$$\iint_R 12 - x^2 - y^2 \, dA$$

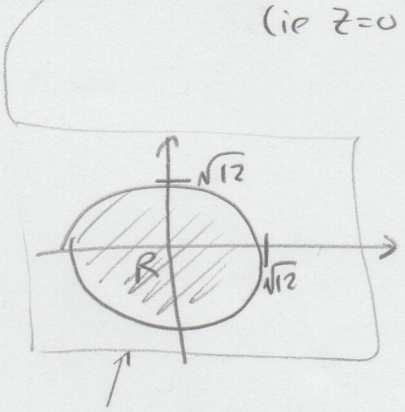
$$= \int_0^{2\pi} \int_0^{\sqrt{12}} (12 - r^2)\, r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{12}} (12r - r^3) \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[6r^2 - \frac{1}{4} r^4 \right]_0^{\sqrt{12}} d\theta$$

$$= \int_0^{2\pi} \left[72 - \frac{1}{4}(144) - 0 \right] d\theta = \int_0^{2\pi} 36 \, d\theta$$

$$= \boxed{72\pi}$$



$z = 12 - x^2 - y^2$ & $z = 0$

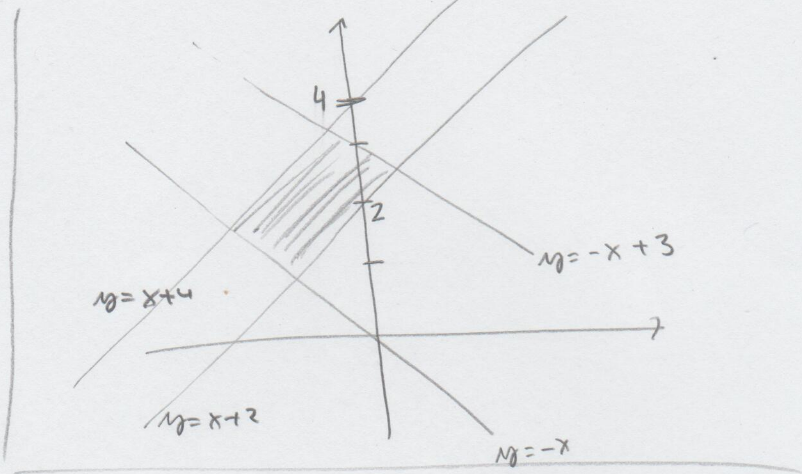
$0 = 12 - x^2 - y^2$

$12 = x^2 + y^2$

(19) $\iint \sin(x-y)\cos(x+y)\, dA$

$y = x+2$ $\qquad$ $y = -x$

$y = x+4$ $\qquad$ $y = -x+3$



① Change of variables

$$\begin{cases} u = x - y \\ v = x + y \end{cases}$$

② Find INVERSE CHANGE of VARIABLE

solve for x, y

$u + v = 2x \Rightarrow x = \frac{1}{2}u + \frac{1}{2}v$

$u = x - y$

$u = \frac{1}{2}u + \frac{1}{2}v - y \Rightarrow y = -\frac{1}{2}u + \frac{1}{2}v$

$$\begin{cases} x = \frac{1}{2}u + \frac{1}{2}v \\ y = \frac{-1}{2}u + \frac{1}{2}v \end{cases}$$

③ Compute the Jacobian

$$J(u,v) = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix} = \frac{1}{4} - \frac{-1}{4} = \frac{1}{2}$$

So $\quad dx\,dy = J(u,v)\,du\,dv$

$= \frac{1}{2}\,du\,dv$

④ Find REGION in u, v. Our four lines

$\underline{y = x+2}$

$\downarrow$

$-\frac{1}{2}u + \frac{1}{2}v = \frac{1}{2}u + \frac{1}{2}v + 2$

$\downarrow$

$u = -2$

$\underline{y = x+4}$

$0 = x - y + 4$

$0 = u + 4$

$u = -4$

$\underline{y = -x}$

$\downarrow$

$0 = x + y$

$\underline{0 = V}$

$\underline{y = -x+3}$

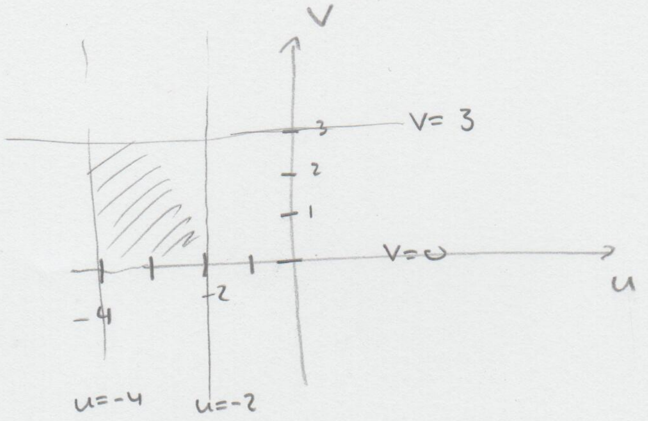
$x + y = 3$

$\underline{\underline{V = 3}}$

19 CONTINUED ⑤ SETUP u, v region

$-4 \le u \le -2$

$0 \le v \le 3$



⑥ INTEGRATE

$$\iint \sin(x-y)\cos(x+y)\,dx\,dy = \iint \sin(u)\cos(v)\,\overbrace{\tfrac{1}{2}\,du\,dv}^{J(u,v)\,du\,dv}$$

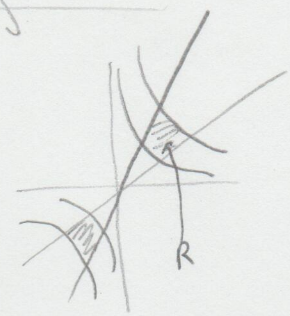
$$= \frac{1}{2}\int_{-4}^{-2} \sin(u)\,du \cdot \int_0^3 \cos(v)\,dv = \frac{1}{2}\cos(u)\Big|_{-4}^{-2} \cdot \sin(v)\Big|_0^3$$

$$\boxed{= \frac{1}{2}\left[\cos(-2) - \cos(-4)\right] \cdot \left[\sin(3) - \sin(0)\right]}$$

(21.) $\iint_R xy \, dA$ $\quad xy=1, \; xy=3 \quad y=x \quad y=3x \quad x=u/v, \; y=v$

① Change of variables

② & inverse change of variables given



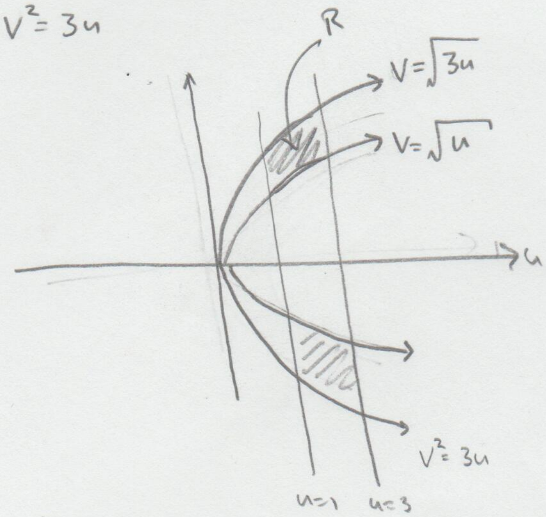
$$\begin{cases} x = u/v = uv^{-1} \\ y = v \end{cases}$$

③ Jacobian

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1/v & -uv^{-2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$$

④ Find Region in u,v. Our functions

$xy=1$	$xy=3$	$y=x$	$y=3x$
$\frac{u}{v}\cdot v=1$	$\frac{u}{v}\cdot v=3$	$v=\frac{u}{v}$	$v=3\frac{u}{v}$
$u=1$	$u=3$	$v^2=u$	$v^2=3u$



⑤ Set up integral

$1 \le u \le 3$

$\sqrt{u} \le v \le \sqrt{3u}$

⑥ Integrate

$$\int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} \frac{u}{v}\cdot v \; \frac{1}{v} \, dv\, du = \int_1^3 \left[\int_{\sqrt{u}}^{\sqrt{3u}} \frac{u}{v} \, dv\right] du$$

$$= \int_1^3 u \cdot \ln|v| \Big|_{\sqrt{u}}^{\sqrt{3u}} du = \int_1^3 u\cdot\left[\ln\sqrt{3u} - \ln\sqrt{u}\right] du = \frac{1}{2}\int u\cdot\left[\ln(3)+\ln(u)-\ln(u)\right] du$$

$$= \frac{1}{2}\int_1^3 \ln(3)\cdot u \, du = \frac{\ln(3)}{2}\,\frac{u^2}{2}\Big|_1^3 = \boxed{\frac{\ln(3)}{4}(9-1)}$$

$u\cdot\left[\frac{1}{2}\ln(3u) - \frac{1}{2}\ln(u)\right]$

$\frac{1}{2}u\left[\ln(3)+\ln(u)-\ln(u)\right]$