Show work. Your work is part of your answer. Yuo may use a calculator.

Name: ______________________________

1. Compute the amount we will have in 3 years if we earn interest compounded annually on $200 at 4% APR.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$= 200\left(1 + \frac{.04}{1}\right)^{1 \cdot 3} = \$224.97$$

2. If we invest $200 at 4% APR (compunded continuously) how long before we have $400.

$$A = Pe^{rt}$$

$$400 = 200\, e^{0.04\, t}$$

$$2 = e^{0.04 t}$$

$$\frac{\ln(2)}{0.04} = t$$

So $\underline{t = 17.3 \text{ years}}$

3. Assume I deposit \$1000 per week for 35 years (assume 52 weeks per year) in my retirement fund. Assume the retirement fund earns 8% per year on average. That is,

$$WPR = (1.08)^{1/52} - 1 = 0.001481115792$$

where WPR is the weekly percentage rate. How is the account worth in 35 years?

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

35 years × 52 weeks per year

$$= 1000 \cdot \frac{(1 + 0.00148..)^{35 \cdot 52} - 1}{0.00148}$$

$$= \$9,310,000$$

4. Write the following system of linear eqations as an augmented matrix.

$$\begin{cases} 2x & -2y & = 8 \\ & y & = 2 \end{cases}$$

$$\left[\begin{array}{cc:c} 2 & -2 & 8 \\ 0 & 1 & 2 \end{array}\right] \quad \begin{array}{c} \frac{1}{2}R_1 \\ R_2 \end{array} \left[\begin{array}{cc:c} 1 & -1 & 4 \\ 0 & 1 & 2 \end{array}\right]$$

$$\begin{array}{c} R_1 + R_2 \\ R_2 \end{array} \left[\begin{array}{cc:c} 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right]$$

5. Solve the previous system of linear equations by Guass Jordan elimination. Reduce theh augemented matrix to RREF.

$(6,2)$

6. Let
$$A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \text{and } \mathbf{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

(a) Compute AC
(b) Compute CA
(c) Compute $C\mathbf{x}$
(d) Compute C^{-1}
(e) Compute $C^{-1}\mathbf{b}$

(a) $\underset{2\times 3}{\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 3 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}}$

NOT POSSIBLE

(b) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} (1)(+1)+(2)(-1) & 4 & 9 \\ 1 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & 9 \\ 1 & 0 & 3 \end{bmatrix}$

(c) $C\vec{x} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y \\ x \end{bmatrix}$

$= -\frac{1}{2}$

(d) $C^{-1} = \left(\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \right)^{-1} = \frac{1}{1\cdot 0 - 2\cdot 1} \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1/2 & -1/2 \end{bmatrix}$

(e) $C^{-1}\vec{b} = \begin{bmatrix} 0 & 1 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

7. Write the linear system below as a matrix multiplication problem as $A\mathbf{X} = \mathbf{b}$.

$$\begin{cases} 2x + 4y = 0 \\ 3x + 5y = 1 \end{cases}$$

(a) Identify A, $\mathbf{b}$, and $\mathbf{x}$.

(b) Compute A^{-1} and $A^{-1}\mathbf{b}$.

(c) Write down the solution to the system.

(a) $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

(b) $A^{-1} = \dfrac{1}{10-12} \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -5/2 & 2 \\ 3/2 & -1 \end{bmatrix}$

$A^{-1}\vec{b} = \begin{bmatrix} -5/2 & 2 \\ 3/2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(c) $(2, -1)$

8. An economy is based on two industrial sectors, coal and steel. Production of a dollar worth of coal requires an input of \$0.10 from the coal sector and \$0.20 from the steel sector. Production of a dollar worth of steel requires an input of \$0.30 from the coal sector and \$0.50 from the steel sector. Find the output for each sector that is needed to satisfy a final demand of \$10 billion for coal and \$30 billion for steel.

$$M = \begin{bmatrix} .1 & .3 \\ .2 & .5 \end{bmatrix} \qquad D = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$$

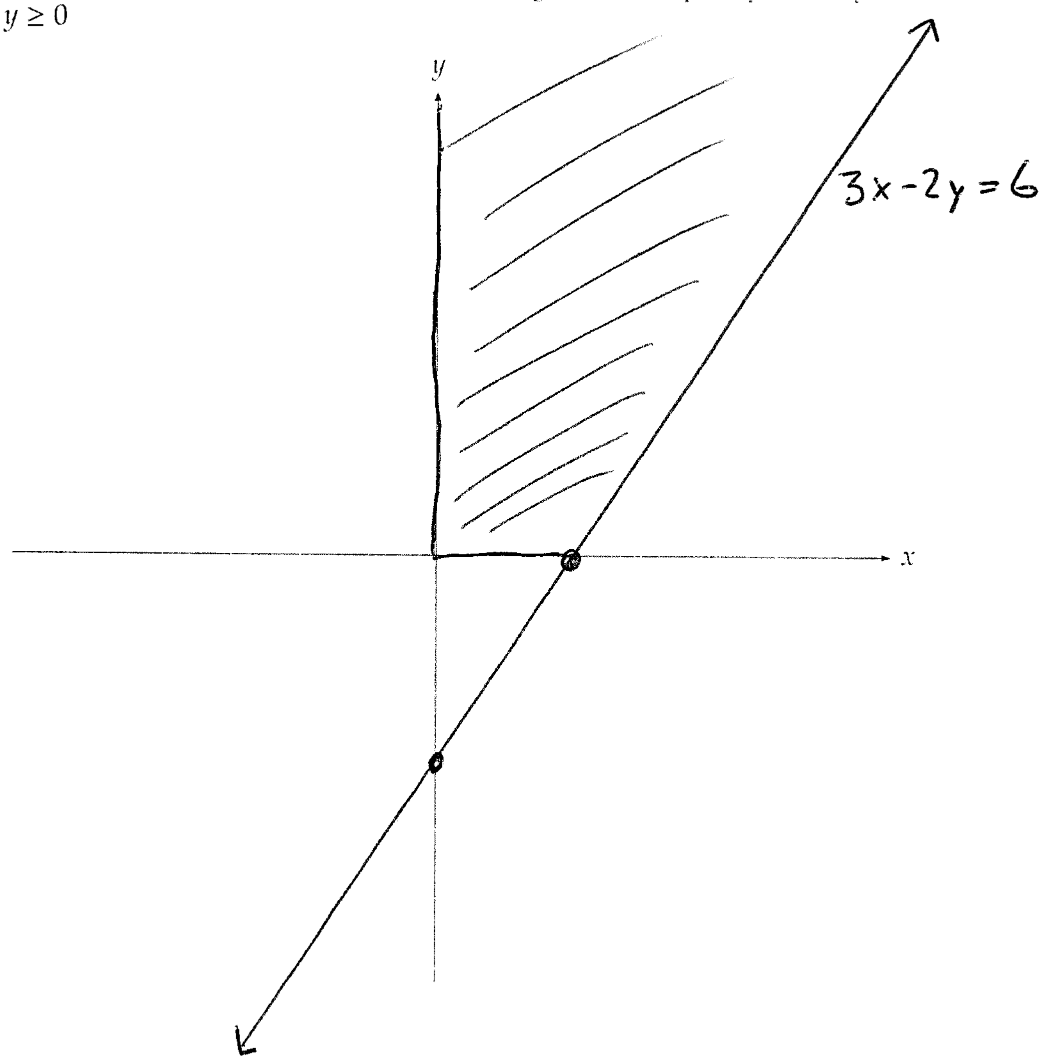
$$X = (I-M)^{-1} D = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .1 & .3 \\ .2 & .5 \end{bmatrix} \right)^{-1} \begin{bmatrix} 10 \\ 30 \end{bmatrix}$$

$$= \begin{bmatrix} .9 & -.3 \\ -.2 & .5 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \frac{1}{(.9)(.5) - (-.3)(-.2)} \begin{bmatrix} .5 & .3 \\ .2 & .9 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \end{bmatrix}$$

$$= \frac{1}{.39} \begin{bmatrix} .5 & .3 \\ .2 & .9 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \frac{1}{.39} \begin{bmatrix} 5 + 9 \\ 2 + 27 \end{bmatrix} = \begin{bmatrix} 14/.39 \\ 29/.39 \end{bmatrix}$$

$$= \begin{bmatrix} 35.9 \\ 74.4 \end{bmatrix}$$

9. Graph the feasible region for the following linear inequality. $3x - 2y \le 6$, $x \ge 0$ and $y \ge 0$



$3x - 2y = 6$

$x=0$ $\left[\begin{array}{l} 3(0) - 2y = 6 \\ y = -3 \end{array}\right. \Rightarrow (0, -3)$

$y=0$ $\left[\begin{array}{l} 3x - 2(0) = 6 \\ x = 2 \end{array}\right. \Rightarrow (2, 0)$