1. Write as an augmented matrix. Row reduce (to RREF). Interpret your result.

$$\begin{cases} x & +y & +z & = 0 \\ 3x & & -z & = 7 \\ x & +2y & & = 7 \end{cases}$$

1 Section 4.7 Leontief Input output matrices

$$X = (I - M)^{-1}D$$

2. An economy is based on two industrial sectors, coal and steel. Production of a dollar worth of coal requires an input of \$0.10 from the coal sector and \$0.20 from the steel sector. Production of a dollar worth of steel requires an input of \$0.20 from the coal sector and \$0.40 from the steel sector. Find the output for each sector that is needed to satisfy a final demand of \$20 billion for coal and \$10 billion for steel.

3. An economy is based on two sectors, coal and steel. Given the technology matrix M and the final demand matrix D (in billions of dollars), find $(I - M)^{-1}$ and the output matrix X:

$$M = \begin{bmatrix} 0.45 & 0.65 \\ 0.55 & 0.35 \end{bmatrix} D = \begin{bmatrix} 40 \\ 10 \end{bmatrix}$$

2 Section 5.1 Linear Inequalities

4. Graph the feasible region for the following linear inequalities

   (a) $y \leq 3x + 1$, $x \geq 0$ and $y \geq 0$

   (b) $x + 3y \geq 9$, $x \geq 0$ and $y \geq 0$

   (c) $y \leq 3x + 1$, $x \geq 0$ and $y \geq 0$

5. Enrollment in finite mathematics plus enrollment in calculus is less than 300.

6. Seed costs for a farmer are \$40 per acre for corn and \$32 per acre for soybeans. How many acres of each crop should the farmer plant if he wants to spend no more than \$5,000 on seed?

7. Labor costs for a farmer are \$55 per acre for corn and \$45 per acre for soybeans. How many acres of each crop should the farmer plant if he wants to spend no more than \$6,900 on labor?

3 Section 5.2-3 Feasible Regions and Linear Optimization

8. Graph the feasible region and find the corner points for the following. Is the region bounded or unbounded?

$$\begin{cases} x & +2y & \leq 10 \\ 3x & +y & \leq 15 \\ \\ x & & \geq 0 \\ & y & \geq 0 \end{cases}$$

9. Graph the feasible region and find the corner points for the following. Is the region bounded or unbounded?

$$\begin{cases} x & +4y & \leq 30 \\ 3x & +y & \leq 24 \\ \\ x & & \geq 0 \\ & y & \geq 0 \end{cases}$$

10. Graph the feasible region and find the corner points for the following. Is the region bounded or unbounded? Maximize and minimize the function $z = x - y$ subject to

$$\begin{cases} x & -2y & \leq -6 \\ 2x & -y & \leq 0 \\ \\ x & & \geq 0 \\ & y & \geq 0 \end{cases}$$

11. Graph the feasible region and find the corner points for the following. Is the region bounded or unbounded? Maximize and minimize the function $P = 3x + 5y$ subject to

$$\begin{cases} x & +2y & \leq 6 \\ x & y & \leq 4 \\ 2x & 3y & \geq 12 \\ \\ x & & \geq 0 \\ & y & \geq 0 \end{cases}$$

12. Transportation. The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs \$1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and costs \$100 to rent. Since there are 400 students in the senior class that may be eligible to go on the trip, the officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent

in order to minimize the transportation costs? What are the minimal transportation costs?

4 Section 6.1-6.2 Simplex Method

13. Use the Simplex method to solve. Maximize $P = x_1 + x_2 + 2x_3$ subject to

$$\begin{cases} x_1 - 2x_2 + x_3 \leq 9 \\ 2x_1 + x_2 + 2x_3 \leq 28 \\ \\ x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \end{cases}$$

14. Use the Simplex method to solve. Maximize $P = 2x_1 + 3x_2$ subject to

$$\begin{cases} -x_1 + x_2 \leq 2 \\ x_2 \leq 4 \\ \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

15. Use the Simplex method to solve. Maximize $P = 4x_1 + 2x_2 + 3x_3$ subject to

$$\begin{cases} x_1 + x_2 + x_3 \leq 11 \\ 2x_1 + 3x_2 + x_3 \leq 20 \\ x_1 + 3x_2 + 2x_3 \leq 20 \\ \\ x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \end{cases}$$

5 Section 7.1-7.4 Logic, Sets, and Counting

16. Solve in two ways: (A) using a tree diagram, and (B) using the multiplication principle.

    (a) How many ways can 2 coins turn up—heads, H, or tails, T—if the combined outcome (H, T) is to be distinguished from the outcome (T, H)?

    (b) How many 2-letter code words can be formed from the first 3 letters of the alphabet if no letter can be used more than once?

17. How many subsets does each of the following sets contain?

    (a) $\{a\}$

    (b) $\{a, b\}$

(c) $\{a, b, c\}$

(d) $\{a, b, c, d\}$

18. Let the universal set U be the set of all 120 students in the class, A the set of students from the College of Arts & Sciences, B the set of students from the College of Business, F the set of freshmen, and S the set of sophomores.

	Freshmen	Sophomores
Arts & Sciences	19	14
Business	66	21

What are the following?

(a) $n(F)$

(b) $n(A)$

(c) $n(A \cap S)$

(d) $n(B \cap F)$

(e) $n(A \cup S)$ How many students are not freshman nor Business students?

19. Committee selection. A company president and three vice-presidents, denoted by the set $\{P, V_1, V_2, V_3\}$, wish to select a committee of 2 people from among themselves. How many ways can this committee be formed? That is, how many 2-person subsets can be formed from a set of 4 people?

20. Voting coalition. The company's leaders in Problem 19 decide for or against certain measures as follows: The president has 2 votes and each vice-president has 1 vote. Three favorable votes are needed to pass a measure. List all minimal winning coalitions; that is, list all subsets of $\{P, V_1, V_2, V_3\}$ that represent exactly 3 votes.

A coin is tossed with possible outcomes of heads H, or tails T. Then a single die is tossed with possible outcomes 1, 2, 3, 4, 5, or 6. How many combined outcomes are there?

21. In how many ways can 3 coins turn up -heads H, or tails T- if combined outcomes such as (H, T, H), (H, H, T), and (T, H, H) are considered as different?

22. An entertainment guide recommends 6 restaurants and 3 plays that appeal to a couple.

(a) If the couple goes to dinner or a play, but not both, how many selections are possible?

(b) If the couple goes to dinner and then to a play, how many combined selections are possible?

23. A high school football team with 40 players includes 16 players who played offense last year, 17 who played defense last year, and 12 who were not on last year's team. How many players from last year played both offense and defense?

24. Compute

    (a) $7!$

    (b) $5! + 6!$

    (c) $\dfrac{500!}{498!}$

    (d) ${}_{12}C_5$

    (e) ${}_{12}P_5$

25. In the following problems, would you consider the selection to be a permutation, a combination, or neither? Explain your reasoning.

    (a) The university president named 3 new officers: a vice-president of finance, a vice-president of academic affairs, and a vice-president of student affairs.

    (b) The university president selected 2 of her vice-presidents to attend the dedication ceremony of a new branch campus.

    (c) A student checked out 4 novels from the library.

    (d) A student bought 4 books: 1 for his father, 1 for his mother, 1 for his younger sister, and 1 for his older brother.

    (e) A father ordered an ice cream cone (chocolate, vanilla, or strawberry) for each of his 4 children.

26. How many ways can a 3-person subcommittee be selected from a committee of 7 people? How many ways can a president, vice-president, and secretary be chosen from a committee of 7 people?

- Leontief Input output matrices

  $X = (I - M)^{-1}D$