Math for Deep Learning - Homework 01

Name:_____

1. Solve the linear system using row reduction.

$$\begin{cases} 3x & +4y & +z & = 0 \\ & -y & +z & = 2 \\ x & +4y & & = -3 \end{cases}$$

$$\begin{cases} x & +4y & +z & = -4 \\ & -y & +z & = 2 \\ x & & +5z & = 4 \end{cases}$$

$$\begin{cases} x & +4y & +z & = -4 \\ & -y & +z & = 2 \\ x & & +5z & = 0 \end{cases}$$

2. Solve the linear system turning into the matrix equation $A\mathbf{x} = \mathbf{b}$, find the inverse of A and compute $A^{-1}\mathbf{b}$.

$$\begin{cases} x & +4y & +z & = -4 \\ & -y & +z & = 2 \\ x & & +5z & = 0 \end{cases}$$

3. For the following matrices as linear transformations and the following linear transformations find the kernel. Also state their domains and codomains.

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ 2x - y \\ x - y \end{bmatrix}$$

$$T\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x + y + z \\ w - y \\ x + z + w \end{bmatrix}$$

4. Find the eigenvectors and eigenvalues for the following matrices.

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 5 & 0 & 2 \end{bmatrix}$$

5. For the following matrices deteremine if you can diadonalize. If yes then diagonalize it.

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 5 & 0 & 2 \end{bmatrix}$$

6. Find the LU decomposition for the following matrices.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 5 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

7. Use LU decomposition to solve.

$$\begin{cases} x & +y & +z & = 1 \\ 4x & +3y & -z & = 6 \\ 3x & +5y & +3z & = 4 \end{cases}$$

8. Find the Gram-Schmidt orthogonalization of the following vestors using the usual inner product.

$$\mathbf{v_1} = (0, 0, 3), \mathbf{v_2} = (1, 0, -2), \mathbf{v_3} = (1, 2, 3)$$

9. Find the Gram-Schmidt orthogonalization of the follwing vectors

$$f_1(x) = 1, f_2(x) = 1 - x, f_3(x) = 1 + x^2$$

with the following inner product.

$$\langle f(x), g(x) \rangle = \int_0^1 f(x)g(x) \, dx$$

10. Find the SVD for the following matrix.

$$A = \begin{bmatrix} 1 & 4 \\ 0 & 3 \\ 0 & 3 \end{bmatrix}$$