Math 3160 - Test 2

Name:____

1. Find the parametric equations for the line (in $\mathbb{R}^3$) so that

   (a) the line contains the point $P(0, 1, 1)$ and $Q(1, 0, 0)$.

   (b) the line contains the point $P(-2, 0, 3)$ and is parallel to the vector $(1, 2, 1)$.

2. Cinsider the points $P(2, 2, 2)$, $Q(1, 2, 3)$ and $R(1, -1, 0)$.

   (a) Find the normal equation for the plane so that the plane contains the three points.

   (b) Find the parametric equation for the plane so that the plane contains the thre points.

3. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } 2x + y = 1\}$. Show $(W, +, \cdot)$ is **not** a subspace of $\mathbb{R}^3$.

4. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } 2x + y = 0\}$. Show $(W, +, \cdot)$ is a subspace of $\mathbb{R}^3$.

5. Let $S = \{(0,0,1),(0,1,2),(0,-1,0)\}$. Is S linearly independent?

6. Let $S = \{(0.1.1), (1,1,0), (0,-1,0)\}$. Is $(2,2,2) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2)$?

7. Let $B = \{(1,2,1), (1,2,1), (0,1,2), (0,-1,0)\}$.

   (a) Show B is not a basis for $\mathbb{R}^3$.

   (b) Find a minimal spanning set of elements of B.

8. For the following system of linear equations.

$$\begin{array}{rrrrl} 2x_1 & -2x_2 & +4x_3 & & = 0 \\ & & x_3 & +6x_4 & = 0 \end{array}$$

(a) Find the solution set.

(b) Find a basis for the solution set.

(c) What is the dimension of that solution set?

9. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the formula
$$T(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x+y \\ y+z \\ x-z \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear transformation T.

(b) Find a basis for the null space of A, NULL(A).

extra credit. Define the following linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the formula $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}) = \begin{bmatrix} x_1 \\ 2x_2 \\ 3x_3 \\ 0 \end{bmatrix}$.

(a) Find the matrix, A, to represent the linear transformation T.

(b) Find a basis for the null space of A, NULL(A).

(c) Prove this is a linear Transformation. That is prove that

i. $\forall \mathbf{u}, \mathbf{v} \in \mathbb{R}^4, T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{u})$

ii. $\forall k \in \mathbb{R}, \mathbf{u} \in \mathbb{R}^4, T(k\mathbf{u}) = kT(\mathbf{u})$

(d) What is this linear transformation?