1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

(a)
$$\begin{cases} x_1 & -2x_2 & & & -6x_5 & = 0 \\ & x_2 & +x_3 & +6x_4 & & = 5 \\ & 2x_2 & & +6x_4 & +x_5 & = 4 \\ & x_2 & -x_3 & & +x_5 & = -1 \end{cases}$$

(b)
$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & & x_3 & = 0 \\ x_1 & +x_2 & +2x_3 & = 0 \end{cases}$$

(c)
$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ -x_1 & -x_2 & +3x_3 & = 2 \\ x_1 & -3x_2 & +7x_3 & = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

(a)
$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$$

(b)
$$\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$$

2 Matrices, Determinants, Cramer's Rule

3. Let $A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the determinant of the matices A, B, C and D

(b) Compute D^3 and D^{-1}.

(c) Compute $C^T C$. What kind of matrix is $C^T C$?

4. Solve the following equations for X assuming any matrix has an inverse. Let A, B, C, X be $n \times n$ matrices and let $\mathbf{u}$ be an $n \times 1$ vector.

   (a) $AX = BX - A$

   (b) $AX = 2X - A$

   (c) $A\mathbf{u} = 2\mathbf{u} + B$

5. Solve the following using Cramer's Rule.
$$\begin{cases} x & -2y & = 0 \\ 5x & -2y & = 0 \end{cases}$$
$$\begin{cases} x_1 & -2x_2 & & = 0 \\ x_1 & & +x_3 & = 0 \\ & x_2 & +3x_3 & = 0 \end{cases}$$

3   Linear Transformations

6. For the following transformation. What is A? What is the dimension of the domain? What is the dinmension of the codomain?
$$\begin{aligned} w_1 & = x_1 + x_3 \\ w_2 & = 3x_2 - x_3 \end{aligned}$$

7. For the following transformation (T), what is A? What is the dimension of the domain? What is the dinmension of the codomain? Compute the value of $T(e_2)$
$$\begin{aligned} w_1 & = x_1 + x_3 \\ w_2 & = 3x_2 - x_3 \end{aligned}$$

8. For the following matrix as a linear transformation, T, what is the dimension of the domain? What is the dinmension of the codomain? Compute the value of $T(e_1)$
$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

9. What transformation, $T : \mathbb{R}^2 \to \mathbb{R}^3$ has the following properties
$$T(e_1) = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}, T(e_2) = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix},$$