

Show all work and no calculators allowed.

Name: _____

1. Compute the following limits if they exist. If not show why.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{7x + y^2}{x^2 + y^2}$

2. Find and classify extrema for $f(x, y) = -4x^3 + 4xy - 2y^2 + 13$.

3. Find and classify extrema for $f(x, y, z) = x^2 + 2y^2 + z^2$ subject to $x - y + 3z = 1$.

4. $\iint_R y + 2 \, dA$ over the triangle with vertices $A(0, 0)$, $B(4, 0)$ and $C(2, 2)$.

5. $\iint_R \frac{x^2 + y^2}{\arctan(y/x)} dA$ over the region inside the circle $x^2 + y^2 = 4$ outside the circle $x^2 + y^2 = 1$ in the second quadrant.

6. $\iint_R \frac{e^{y-2x}}{y+3x} dA$ over the region defined the lines $y = 2x + 2$, $y = 2x + 4$, $y = -3x + 1$ and $y = -3x + 2$.

7. $\oint_C \langle e^{x^3}, x^2 + y^2 \rangle \cdot d\mathbf{r}$ over the region inside the triangle defined by the points $(0, 0)$ to $(3, 0)$ to $(3, 6)$ and back to $(0, 0)$

Second Derivative Test

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$$

1. If $D(x, y) > 0$ and $f_{xx}(x, y) < 0$ then the function has a Maximum.
2. If $D(x, y) > 0$ and $f_{xx}(x, y) > 0$ then the function has a Minimum.
3. If $D(x, y) < 0$ then the function has a Saddle Point.
4. If $D(x, y) = 0$ then the test is inconclusive.