

Show all work and no calculators allowed.

Name: _____

1. Compute the following limits if they exist. If not show why.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 + y^2}{x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{1 - e^{x^2+y^2}}{x^2 + y^2}$

2. Find and classify extrema for $f(x, y) = x^3 + y^3 - 3x - 12y + 4$.

3. Find and classify extrema for $f(x, y, z) = x + 3y - z$ subject to $x^2 + y^2 + z^2 = 4$.

4. $\iint_R 4yx \, dA$ over the region R defined in the xy plane as under the parabola $y = 4 - x^2$ and above the x -axis

5. $\iint_R e^{x^2+y^2} dA$ over the region inside the circle $x^2 + y^2 = 1$ in the first quadrant.

6. $\iint_R \frac{x+y}{2x-y} dA$ over the region defined the lines $y = 2x - 1$, $y = 2x - 4$, $y = -x + 1$ and $y = -x + 2$.

7. $\oint_C \langle 6x^3 - 2x + y, x + 3x^2 \rangle \cdot d\mathbf{r}$ over the region inside the triangle defined by the points $(0, 0)$ to $(3, 3)$ to $(3, -6)$ and back to $(0, 0)$. Notice the path is clockwise.

Second Derivative Test

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$$

1. If $D(x, y) > 0$ and $f_{xx}(x, y) < 0$ then the function has a Maximum.
2. If $D(x, y) > 0$ and $f_{xx}(x, y) > 0$ then the function has a Minimum.
3. If $D(x, y) < 0$ then the function has a Saddle Point.
4. If $D(x, y) = 0$ then the test is inconclusive.