1 Paths

1. Let $\mathbf{r}(t) = \langle t^2 - t, t^3 - 3t^2 + 3 \rangle$

   (a) Find the position, velocity and acceleration of the particle at time $t = 2$.

   (b) Graph the position, velocity and acceleration appropriately.

2. Let $\mathbf{r}(t) = \langle \sin(e^{-t}), \cos(e^{-t}) \rangle$

   (a) Find the speed function
   $$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

   (b) Compute the arc length from $t = 0$ to $t = 1$.

   (c) Compute the arc length from $t = 0$ to $t = \infty$.

   (d) What is the graph of $\mathbf{r}(t)$?

3. Let $\mathbf{r}(t) = \langle e^{-t} \sin(t), e^{-t} \cos(t) \rangle$

   (a) Find the speed function
   $$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

   (b) Compute the arc length from $t = 0$ to $t = 1$.

   (c) Compute the arc length from $t = 0$ to $t = \infty$.

   (d) What is the graph of $\mathbf{r}(t)$?

2 Functions of Several Variables

4. Compute the following limits if they exist. If not show why.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2 + 1}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1 - \cos(x^2 + y^2)}{x^2 + y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2}$

5. Let $f(x, y) = x^2 + y^2$. Consider the points $P(0, 2)$ and $Q(1, 2)$.

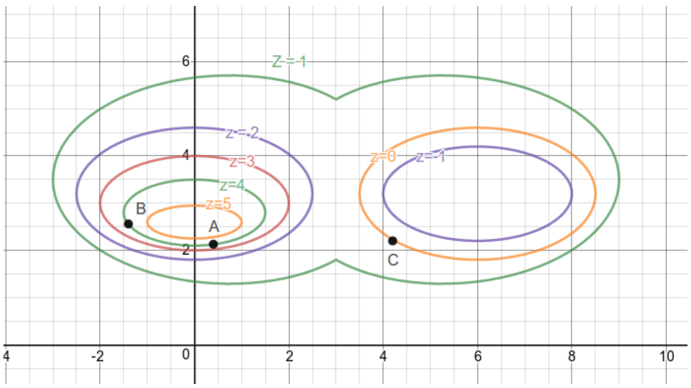
   (a) Graph the contour plot. Include $z = -1, 0, 1, 2, 3, 4$.

   (b) Compute the $\nabla f(x, y)$

   (c) Compute the $\nabla f(P)$ and $\nabla f(Q)$. Also compute their norms.

(d) Graph $\nabla f(P)$ and $\nabla f(Q)$ with initial points P and Q respectively.

6. Draw the gradient at each point A, B and C.



7. Fill in the blanks.

(a) The gradient is the direction of _________ increase.

(b) The gradient is _________ to the contour lines.

(c) The norm of the gradient is _________.

(d) A larger norm of one gradient is _________ graphically.

8. Let $f(x, y) = e^{xy^2+2} - xy^3 + 2$. Find the tangent plane to $f(x, y, z)$ at the point $(-2, 1)$. Use that plane to estimate $f(-2.1, 0.8)$. Compare to the real value of $f(-2.1, 0.8)$.

9. Let $f(x, y, z) = x^2y - xy^2 + z^3$. Find the tangent plane (actually a hyperplane) to $f(x, y, z)$ at the point $(1, 2, 3)$

10. [1] Let $A(1, 2, 3)$ and $B(1, 1, 2)$ be points in $\mathbb{R}^3$. Let $\mathbf{v} = \langle 1, 2, 3\rangle$ and $\mathbf{w} = \langle 2, 2, 4\rangle$ and $f(x, y, z) = x^2 + y^2 + z^2$.

(a) Compute $\nabla f(A)$ and $\|\nabla f(A)\|$.

(b) Compute $D_{\mathbf{v}}f(B)$ and $D_{\mathbf{w}}f(B)$. One is bigger than the other. Interpret.

(c) Compute $D_{\mathbf{v}}f(A)$. Compare to $\|\nabla f(A)\|$.

11. [1] Let $f(x, y) = x^3 + x^2y^2$. Let $x(t) = t^2 + 1$ and $y(t) = 2t - 1$.

(a) Use the chain rule to compute $\frac{df}{dt}$.

(b) Compute $\frac{df}{dt}$ at $t = 1$.

12. Find and classify extremma.

(a) $f(x, y) = x^2 - xy + y^3$.

[1] Not covered, extra credit

(b) $f(x,y) = x^2 + 2xy - y^4$.

(c) $f(x,y,z) = x + 3y + z$ subject to $x^2 + y^2 + z^2 = 1$.

(d) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2$ subject to $x + y + z - 3w = 4$.

(e) $f(x,y,z,w) = x\ln(x) + y\ln(y) + z\ln(z)$ subject to $x + y + z = 1$. In this problem f is called information entropy.

3 Integrals

13. $\iint_R x + y\, dA$ over the region defined by $x + y = 2$ and the coordinate axes.

14. $\iint_R xy\, dA$ over the region defined by $y = x^2$ and the line $y = x + 1$.

15. $\iint_R e^{x^2}\, dA$ over the region defined by $y = -x$, $y = 2x$ and the vertical line $x = 4$.

16. $\iint_R e^{x^2+y^2}\, dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ in the third quadrant.

17. $\iint_R \sqrt{\frac{\tan^{-1}(y/x)}{x^2+y^2}}\, dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ above the lines $y = -x$ and $y = x$.

18. Find the volume below the paraboloid $z = 12 - x^2 - y^2$ and above the xy-plane.

19. $\iint_R \sin(x-y)\cos(x+y)\, dA$ over the region defined the lines $y = x + 2$, $y = x + 4$, $y = -x$ and $y = -x + 3$. Hint the change of variables is $u = x - y$ and $v = x + y$.

20. $\iint_R \frac{x-y}{2x+y}\, dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -2x + 2$ and $y = -2x + 3$.

21. $\iint_R xy\, dA$ over the region defined the graphs of $xy = 1$, $xy = 3$ and the lines $y = x$ and $y = 3x$. Hint $x = u/v$ and $y = v$.

22. $\iint_R (x-y)e^{x^2-y^2}\, dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -x$ and $y = -x + 3$.

23. $\iint_R e^{x^2+4y^2}\, dA$ over the region defined by the portion of the ellipse $\frac{x^2}{4} + y^2 = 1$ in the third quadrant. Hint use the change of variables $x = 2v\cos(u)$ and $x = v\sin(u)$. And note I had $\pi \leq u \leq \frac{3\pi}{2}$

4 Line Integrals

24. $\int_C x\,dx$. Let C be line segment from $(0,1)$ to $(3,2)$.

25. $\int_C xy\,ds$. Let C be line segment from $(0,1)$ to $(3,2)$.

26. $\int_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be line segment from $(0,1)$ to $(3,2)$.

27. $\int_C x\,dy$. Let C be line segment from $(0,1)$ to $(3,2)$.

28. $\oint_C xy\,dx$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

29. $\oint_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

30. $\oint_C \langle 1, xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2+y^2=4$ traced counter-clockwise.

31. $\oint_C -x+yds$. Let C be the circle $x^2+y^2=4$ traced counter-clockwise.

5 Green's Theorem

32. $\oint_C \langle x, -y\rangle \cdot d\mathbf{r}$. Let C be outside of the rectangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ to $(1,0)$ and then back to $(0,0)$.

33. $\oint_C \langle e^{x^3}-xy, e^{y^3}-y\rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

34. $\oint_C \langle \cos(x^2)+y, \cos(y^2)+xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2+y^2=4$ traced counter-clockwise.

6 Div/Grad/Curl

35. Define

$$f(x,y,z)=x^3-yz^2 \text{ and } \mathbf{F}(x,y,z)=\langle x^3, yz^2, xy\rangle.$$

Compute the following, if possible, and if not possible state why.

(a) $\text{div}(f(x, y, z))$

(b) $\text{grad}(f(x, y, z))$

(c) $\text{curl}(f(x, y, z))$

(d) $\text{div}(\mathbf{F}(x, y, z))$

(e) $\text{grad}(\mathbf{F}(x, y, z))$

(f) $\text{curl}(\mathbf{F}(x, y, z))$

(g) $\nabla \cdot \mathbf{F}(x, y, z)$

(h) $\nabla \times (\nabla \cdot \mathbf{F}(x, y, z))$

(i) $\nabla \times (\nabla f(x, y, z))$