Math 2320 - Final Exam Review Version 2

Hint/. Six problems guaranteed to be on the final exam, one integral by definitin, plus one of each of technique of integration (u-sub, byparts, trig integral, trig sub and partial fractions).

1 Definition of the integral

1. Write the definition of the integral.
2. Using the definition of the integral compute

$$\int_1^4 3x^2\,dx$$

2 Applications of the integral

Some good functions to be able to graph here area lines, parabolas, $y = e^x$, $y = \ln(x)$, the trig functions and their translations (such as $y = -2x^2 + 1$). Later you should know the conic sections.

3. Let $a(t) = t^2, v(0) = 0$ and $s(0) = 16$.
   (a) Find $v(t)$ and $s(t)$.
   (b) When is the particle at rest?
   (c) When is the particle at ground height?
4. Find the area between $y = e^x$, $y = \frac{1}{2}x$, $y = 1$ and $y = e$.
5. Revolve the region about the x-axis and compute its volume. The region is inside of $y = x^2$, and $y = 2x$.
6. Revolve the region about the x-axis and compute its volume. The region is inside of $x = y^2$, and $x = 9$.
7. Revolve the region about the x-axis and compute its volume. The region is inside of $x = y^2$, and $y = x - 2$.

3 Techniques of integration

8. $\int xe^{x^2}\,dx$
9. $\int x\sqrt{x^2+1}\,dx$
10. $\int \frac{x}{x^2+1}\,dx$
11. $\int \frac{1}{x^2+1}\,dx$
12. $\int [tan(3x)+1]^{1/3}\sec^2(3x)\,dx$
13. $\int \frac{\sqrt{\ln(x)+1}}{x}\,dx$
14. $\int \frac{\sqrt{\ln(x)+1}}{x}\,dx$
15. $\int xe^{2x}\,dx$
16. $\int x\sin(3x)\,dx$
17. $\int \arctan(x)\,dx$ Hint: $u = \arctan(x)$ and $dv = 1\,dx$
18. $\int x^2e^x\,dx$
19. $\int x^2\ln(x)\,dx$
20. $\int x\ln(x+1)\,dx$
21. $\int \sin^2(x)\,dx$
22. $\int \sin^3(x)\,dx$

23. $\int \cos^2(x)\,dx$

24. $\int \cos^2(x)\sin(x)\,dx$

25. $\int \frac{1}{\sqrt{4x^2-9}}\,dx$ Hint. No triangle needed.

26. $\int \frac{1}{(4-x^2)^{3/2}}\,dx$

27. $\int \frac{1}{(x^2+25)^{3/2}}\,dx$

28. $\int \frac{1}{x^2-4}\,dx$

29. $\int \frac{1}{x^3-x^2}\,dx$

30. $\int \frac{1}{x^4-x^2}\,dx$

31. $\int \frac{2x^2+3x+3}{(x+2)(x^2+1)}\,dx$

4 Sequences and Series

32. $\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$

33. $\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$

34. $\sum_{k=1}^{\infty} \sqrt{k+1} - 2\sqrt{k} + \sqrt{k-1}$

35. $\sum_{k=-2}^{\infty} \frac{1}{3^n}$

36. $\sum_{k=-2}^{\infty} \frac{1}{3^{-n}}$

37. $\sum_{k=0}^{\infty} \frac{7^n}{3^n}$

38. $\sum_{k=1}^{\infty} \frac{k+2}{2k+3}$

39. $\sum_{k=1}^{\infty} \left(1 + \frac{1}{2k+3}\right)^k$

40. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use integral test

41. $\sum_{k=7}^{\infty} \frac{1}{k\ln(k)}$ Use integral test

42. $\sum_{k=1}^{\infty} \frac{1}{k^2}$ Use p-Series test

43. $\sum_{k=1}^{\infty} \frac{1}{k}$ Use p-Series test

44. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ Use p-Series test

45. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use LCT

46. $\sum_{k=1}^{\infty} \frac{3k}{k^3+1}$

47. $\sum_{k=1}^{\infty} \frac{\sqrt{k^2+5}}{k^2+1}$

48. $\sum_{k=1}^{\infty} (-1)^n \frac{k}{k^2+1}$

49. $\sum_{k=1}^{\infty} (-1)^n \left(1 - \frac{1}{k}\right)^k$

50. $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ Use Ratio test

51. $\sum_{k=1}^{\infty} \frac{2^k}{e^k + 1}$

52. $\sum_{k=1}^{\infty} \frac{k^2}{e^k}$

53. $\sum_{k=1}^{\infty} \frac{1}{k^k}$

54. $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

55. $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

56. $\sum_{k=1}^{\infty} (\frac{k}{2k+1})^k$

57. $\sum_{k=1}^{\infty} (\frac{3k^2+1}{2k^2+17k+91})^k$

58. $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^{k^2}$

5 Power and Taylor Series

59. What are the radii and intervals of convergence of the following power series.

    (a)

    (b) $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

    (c)

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

(e)

(f) $\sum_{n=1}^{\infty} \frac{1}{n^2} (x-5)^n$

(g)

(h) $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$

(i)

(j) $\sum_{n=0}^{\infty} x^n$

(k)

(l) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

60. Find the Taylor series from the definition

    (a) $f(x) = \sin(2x)$ at $x = 0$

    (b) $f(x) = e^x$ at $x = 1$

    (c) $f(x) = \frac{1}{1-x}$ at $x = 0$

    (d) $f(x) = \ln(x)$ at $x = 1$

61. Find the Taylor series from a known Taylor series. Using only the taylor series for e^x, $\sin(x)$, $\cos(x)$, and $\frac{1}{1-x}$

    (a) $f(x) = x \sin(x^2) - x^3$

    (b) $f(x) = \frac{\sin(x)}{x}$

    (c) $f(x) = \frac{e^{x^2} - 1 - x^2}{x^4}$

6 Parametric Equations

62.

    (a)

    (b)

(c)

63.

(a)

(b)

(c)

64.

65.

7 Polar Coordinates

66. Graph the following polar equations.

(a) $r = 3$

(b) $r = 4\sin(\theta)$

(c) $r = \sin(2\theta)$

(d) $r = 1 + 2\sin(\theta)$

67.

8 Conic Sections

68. Graph the given conic sections.

(a) $x^2 + \frac{y^2}{4} = 1$

(b) $x^2 - \frac{y^2}{4} = 1$

(c) $x + \frac{y^2}{4} = 1$

(d) $x^2 + 4y^2 = 1$

(e) $(x-1)^2 - \frac{y^2}{4} = 1$