

Hint/. Six problems guaranteed to be on the final exam, one integral by definition, plus one of each of technique of integration (u-sub, byparts, trig integral, trig sub and partial fractions).

1 Definition of the integral

1. Write the definition of the integral.
2. Using the definition of the integral compute

$$\int_1^4 3x^2 dx$$

2 Applications of the integral

Some good functions to be able to graph here area lines, parabolas, $y = e^x$, $y = \ln(x)$, the trig functions and their translations (such as $y = -2x^2 + 1$). Later you should know the conic sections.

3. Let $a(t) = t^2$, $v(0) = 0$ and $s(0) = 16$.
 - (a) Find $v(t)$ and $s(t)$.
 - (b) When is the particle at rest?
 - (c) When is the particle at ground height?
4. Find the area between $y = e^x$, $y = \frac{1}{2}x$, $y = 1$ and $y = e$.
5. Revolve the region about the x -axis and compute its volume. The region is inside of $y = x^2$, and $y = 2x$.
6. Revolve the region about the x -axis and compute its volume. The region is inside of $x = y^2$, and $x = 9$.
7. Revolve the region about the x -axis and compute its volume. The region is inside of $x = y^2$, and $y = x - 2$.

3 Techniques of integration

8. $\int x e^{x^2} dx$

9. $\int x\sqrt{x^2+1} \, dx$
10. $\int \frac{x}{x^2+1} \, dx$
11. $\int \frac{1}{x^2+1} \, dx$
12. $\int [\tan(3x)+1]^{1/3} \sec^2(3x) \, dx$
13. $\int \frac{\sqrt{\ln(x)+1}}{x} \, dx$
14. $\int \frac{\sqrt{\ln(x)+1}}{x} \, dx$
15. $\int xe^{2x} \, dx$
16. $\int x \sin(3x) \, dx$
17. $\int \arctan(x) \, dx$ Hint: $u = \arctan(x)$ and $dv = 1 \, dx$
18. $\int x^2 e^x \, dx$
19. $\int x^2 \ln(x) \, dx$
20. $\int x \ln(x+1) \, dx$
21. $\int \sin^2(x) \, dx$
22. $\int \sin^3(x) \, dx$
23. $\int \cos^2(x) \, dx$
24. $\int \cos^2(x) \sin(x) \, dx$

$$25. \int \frac{1}{\sqrt{4x^2 - 9}} dx \text{ Hint. No triangle needed.}$$

$$26. \int \frac{1}{(4 - x^2)^{3/2}} dx$$

$$27. \int \frac{1}{(x^2 + 25)^{3/2}} dx$$

$$28. \int \frac{1}{x^2 - 4} dx$$

$$29. \int \frac{1}{x^3 - x^2} dx$$

$$30. \int \frac{1}{x^4 - x^2} dx$$

$$31. \int \frac{2x^2 + 3x + 3}{(x + 2)(x^2 + 1)} dx$$

4 Sequences and Series

$$32. \sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$$

$$33. \sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$$

$$34. \sum_{k=1}^{\infty} \sqrt{k+1} - 2\sqrt{k} + \sqrt{k-1}$$

$$35. \sum_{k=-2}^{\infty} \frac{1}{3^n}$$

$$36. \sum_{k=-2}^{\infty} \frac{1}{3^{-n}}$$

$$37. \sum_{k=0}^{\infty} \frac{7^n}{3^n}$$

38. $\sum_{k=1}^{\infty} \frac{k+2}{2k+3}$
39. $\sum_{k=1}^{\infty} \left(1 + \frac{1}{2k+3}\right)^k$
40. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use integral test
41. $\sum_{k=7}^{\infty} \frac{1}{k \ln(k)}$ Use integral test
42. $\sum_{k=1}^{\infty} \frac{1}{k^2}$ Use p-Series test
43. $\sum_{k=1}^{\infty} \frac{1}{k}$ Use p-Series test
44. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ Use p-Series test
45. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ Use LCT
46. $\sum_{k=1}^{\infty} \frac{3k}{k^3+1}$
47. $\sum_{k=1}^{\infty} \frac{\sqrt{k^2+5}}{k^2+1}$
48. $\sum_{k=1}^{\infty} (-1)^n \frac{k}{k^2+1}$
49. $\sum_{k=1}^{\infty} (-1)^n \left(1 - \frac{1}{k}\right)^k$
50. $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ Use Ratio test

$$51. \sum_{k=1}^{\infty} \frac{2^k}{e^k + 1}$$

$$52. \sum_{k=1}^{\infty} \frac{k^2}{e^k}$$

$$53. \sum_{k=1}^{\infty} \frac{1}{k^k}$$

$$54. \sum_{k=1}^{\infty} \frac{k!}{k^k}$$

$$55. \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$56. \sum_{k=1}^{\infty} \left(\frac{k}{2k+1} \right)^k$$

$$57. \sum_{k=1}^{\infty} \left(\frac{3k^2 + 1}{2k^2 + 17k + 91} \right)^k$$

$$58. \sum_{k=1}^{\infty} \left(1 - \frac{1}{k} \right)^{k^2}$$

5 Power and Taylor Series

59. What are the radii and intervals of convergence of the following power series.

(a)

(b) $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

(c)

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

(e)

$$(f) \sum_{n=1}^{\infty} \frac{1}{n^2} (x-5)^n$$

(g)

$$(h) \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

(i)

$$(j) \sum_{n=0}^{\infty} x^n$$

(k)

$$(l) \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

60. Find the Taylor series from the definition

$$(a) f(x) = \sin(2x) \text{ at } x = 0$$

$$(b) f(x) = e^x \text{ at } x = 1$$

$$(c) f(x) = \frac{1}{1-x} \text{ at } x = 0$$

$$(d) f(x) = \ln(x) \text{ at } x = 1$$

61. Find the Taylor series from a known Taylor series. Using only the Taylor series for e^x , $\sin(x)$, $\cos(x)$, and $\frac{1}{1-x}$

$$(a) f(x) = x \sin(x^2) - x^3$$

$$(b) f(x) = \frac{\sin(x)}{x}$$

$$(c) f(x) = \frac{e^{x^2} - 1 - x^2}{x^4}$$

6 Parametric Equations

~~Find the parametric equation for the following equations given in rectangular coordinates.~~

$y = x^2$ $y = 3x - 1$ $y^2 + y = 2x + 2$ Find equation in rectangular coordinates for the following equations given in the parametrically.

62. ~~63.~~ $x = 3t$ and $y = 2t - 1$ $x = 3t$ and $y = 2t^2 - 1$ $x = \cos(t)$ and $y = 3 \sin(t)$ Graph the following parametric equations

$x = 3t$ and $y = 2t - 1$ $x = 3t$ and $y = 2t^2 - 1$ $x = \cos(t)$ and $y = 3\sin(t)$ $x = t\cos(t)$ and $y = t\sin(t)$
 For the following find the equation of the tangent line at the given point.

$x = 3t$ and $y = 2t - 1$ at $P = (6, 3)$ $x = 3t$ and $y = 2t^2 - 1$ at $t = 1$ $x = \cos(t)$ and $y = 3\sin(t)$ at $t = \pi/4$

7 Polar Coordinates

Graph the following polar equations.

1. $r = 3$
2. $r = 4\sin(\theta)$
3. $r = \sin(2\theta)$
4. $r = 1 + 2\sin(\theta)$

For the following find the equation of the tangent line at the given point.

1. $r = 3$ at $P = (x_0, y_0) = (\frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}})$
2. $r = 4\sin(\theta)$ at $\theta = \pi/4$
3. $r = \sin(2\theta)$ at $\theta = \pi/2$
4. $r = 1 + 2\sin(\theta)$ at $\theta = \pi/3$

8 Conic Sections

Graph the given conic sections.

1. $x^2 + \frac{y^2}{4} = 1$
2. $x^2 - \frac{y^2}{4} = 1$
3. $x + \frac{y^2}{4} = 1$
4. $x^2 + 4y^2 = 1$
5. $(x - 1)^2 - \frac{y^2}{4} = 1$