

PRACTICE TEST 3

$$1. \int \frac{1}{x \ln(x)} dx \quad \left| \begin{array}{l} u = \ln(x) \\ du = \frac{1}{x} dx \end{array} \right.$$

$$= \int \underbrace{\frac{1}{\ln(x)}}_u \cdot \underbrace{\frac{1}{x} dx}_{du} = \int \frac{1}{u} \cdot du = \ln|u| + C = \ln|\ln(x)| + C$$

$$2. \int \frac{1}{x (\ln(x))^2} dx \quad \left| \begin{array}{l} u = \ln(x) \\ du = \frac{1}{x} dx \end{array} \right.$$

$$= \int \frac{1}{u^2} du = \int u^{-2} du = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{\ln(x)} + C$$

$$3. \int \tan(x) dx = \ln|\sec(x)| + C$$

one of the 16

$$4. \int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

$$5. \int \frac{1}{\sqrt{x^2-4}} dx$$

$$\begin{array}{l} x^2 - 4 \\ u = a^2 \\ u = x \quad a = 2 \\ u = 2 \sec(\theta) \end{array}$$

only used to
set up problem
No longer needed

$$x = 2 \sec(\theta)$$

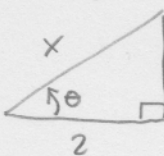
$$dx = 2 \sec(\theta) \tan(\theta)$$

$$\int \frac{1}{\sqrt{x^2-4}} dx = \int \frac{1}{\sqrt{(2\sec(\theta))^2-4}} \cdot 2\sec(\theta) \tan(\theta) d\theta = 2 \int \frac{\sec(\theta) \tan(\theta)}{\sqrt{4\sec^2(\theta)-4}} d\theta$$

$$= 2 \int \frac{\sec(\theta) \tan(\theta)}{\sqrt{4(\sec^2(\theta)-1)}} d\theta = \int \frac{\sec(\theta) \tan(\theta)}{\tan(\theta)} d\theta = \int \sec(\theta) d\theta = \ln|\sec(\theta) + \tan(\theta)| + C$$

TRIANGLE FROM ORIGINAL SUB

$$\frac{x}{2} = \sec(\theta) = \frac{\text{Hyp}}{\text{Adj}}$$



$$\sec(\theta) = \frac{x}{2}$$

$$\tan(\theta) = \frac{\sqrt{x^2-4}}{2} \quad \begin{array}{l} \text{opp} \\ \text{adj} \end{array}$$

$$= \ln \left| \frac{x}{2} + \frac{\sqrt{x^2-4}}{2} \right| + C$$

⑥ $\int \frac{2x+2}{x^2+2x} dx$ use partial fractions

$$\frac{2x+2}{x(x+2)} = \left[\frac{A}{x} + \frac{B}{x+2} \right] x(x+2)$$

$$2x+2 = A(x+2) + Bx$$

$$2x+2 = Ax+2A+Bx$$

$$2x+2 = (A+B)x + 2A$$

$$x: 2 = A+B$$

$$1: 2 = 2A$$

$$\swarrow \text{so } A=1$$

$$\text{AND } B=1$$

$$\int \frac{2x+2}{x^2+2x} dx = \int \frac{1}{x} + \frac{1}{x+2} dx = \boxed{\ln|x| + \ln|x+2| + C}$$

⑦ $\int \frac{2x^2+x+1}{x^3+x} dx$ use partial fractions

$$\frac{2x^2+x+1}{x(x^2+1)} = \left(\frac{A}{x} + \frac{Bx+C}{x^2+1} \right) x(x^2+1)$$

$$2x^2+x+1 = A \cdot (x^2+1) + (Bx+C)x$$

$$2x^2+x+1 = Ax^2 + A + Bx^2 + Cx$$

$$2x^2+x+1 = (A+B)x^2 + Cx + A$$

$$\left. \begin{array}{l} x^2: 2 = A+B \\ x: 1 = C \\ 1: 1 = A \end{array} \right\} \Rightarrow \begin{array}{l} A=1, B=1 \text{ AND} \\ C=1 \end{array}$$

$$\text{So } \int \frac{2x^2+x+1}{x^3+x} dx = \int \frac{1}{x} + \frac{x+1}{x^2+1} dx$$

$$= \int \frac{1}{x} dx + \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx = \ln|x| + \frac{1}{2} \ln|x^2+1| + \arctan(x) + C$$

I used u-sub

here

$$u = x^2+1$$

PARTIAL FRACTIONS

$$\textcircled{8} \int \frac{2x^2 + 3x + 1}{x(x+1)^2} dx \quad \left(\frac{2x^2 + 3x + 1}{x(x+1)^2} = \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \right) \cdot x \cdot (x+1)^2 \right)$$

$$2x^2 + 3x + 1 = A(x+1)^2 + Bx(x+1) + C \cdot x$$

$$= A(x^2 + 2x + 1) + Bx^2 + Bx + Cx$$

$$= (A+B)x + (2A+B+C)x + A$$

$$x^2: 2 = A + B$$

$$x: 3 = 2A + B + C$$

$$1: 1 = A$$

$$A = 1$$

$$2 = 1 + B$$

$$1 = B$$

$$3 = 2(1) + (1) + C$$

$$0 = C$$

So

$$\int \frac{2x^2 + 3x + 1}{x(x+1)^2} dx = \int \left(\frac{1}{x} + \frac{1}{x+1} + \frac{0}{(x+1)^2} \right) dx = \ln|x| + \ln|x+1| + C$$

9, 10 we did in class but I'll do #11

$$\textcircled{11} \int_1^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-1/2} dx = \lim_{b \rightarrow \infty} \left. \frac{x^{1/2}}{1/2} \right|_1^b$$

$$= \lim_{b \rightarrow \infty} [2(b)^{1/2} - 2(1)^{1/2}] = \infty \quad \text{the integral diverges.}$$

(12.) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ } You may use the formula $\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = e^r$
or take the $\ln$ of both sides

(13.) did in class

(14.) $\lim_{x \rightarrow 0^+} (1+x)^{1/x}$ ← Look's similar to #12

$$\ln(L) = \lim_{x \rightarrow 0^+} \ln((1+x)^{1/x})$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(1+x)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1} = \lim_{x \rightarrow 0^+} \frac{1}{1+x} = 1$$

So $\ln(L) = 0 \Rightarrow L = e^1 = e$

(15.) for you but very like #14.

(16.) Did in class

(17.) $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

use telescoping since $\ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln(n)$

$$S_1 = \sum \ln(n+1) - \ln(n) = \ln(2) - \ln(1) = \ln(2)$$

$$S_2 = \sum \ln(n+1) - \ln(n) = (\ln(2) - \ln(1)) + (\ln(3) - \ln(2)) = \ln(3)$$

$$S_3 = \sum \ln(n+1) - \ln(n) = (\ln(2) - \ln(1)) + (\ln(3) - \ln(2)) + (\ln(4) - \ln(3)) = \ln(4)$$

So

$$S_N = \ln(N+1)$$

Note $\lim_{N \rightarrow \infty} S_N = \lim_{N \rightarrow \infty} \ln(N+1) = \infty$

So series diverges

$$(18.) \sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$$

Geometric Series

$$\sum \left(\frac{2}{3}\right)^n = \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \dots$$

$$a = \frac{2}{3}$$

$$r = \frac{2}{3}$$

Since $|r| = |\frac{2}{3}| < 1$, the series converges to $\frac{a}{1-r} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \boxed{2}$.

$$(19.) \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$$

Geometric Series

$$r = \frac{3}{2}$$

$|r| = \frac{3}{2} \geq 1$ So series diverges

$$(20.) \sum_{n=7}^{\infty} e^{-n}$$

Geometric Series

$$= e^{-7} + e^{-8} + e^{-9} + e^{-10} + \dots$$

$$a = e^{-7}$$

$$r = e^{-1} = \frac{1}{e}$$

Since $|r| = \frac{1}{e} < 1$ the series converges to $\frac{a}{1-r} = \frac{e^{-7}}{1-e^{-1}} = \frac{1}{e^7 - e^6}$.

$$(21.) \sum_{n=7}^{\infty} \frac{1}{n \ln(n)}$$

Integral Test

see Problem #1

$$\int_7^{\infty} \frac{1}{x \ln(x)} dx = \lim_{b \rightarrow \infty} \ln \left(\int_7^b \frac{1}{x \ln(x)} dx \right) = \lim_{b \rightarrow \infty} \ln(\ln(x)) \Big|_7^b$$

$$= \lim_{b \rightarrow \infty} \ln(\ln(b)) - \ln(\ln(7)) = \infty$$

Since the integral diverges the series diverges.

$$(22) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$$

Divergence Test

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{e} \neq 0$$

So series diverges

$$(23) \sum \frac{n^2+1}{3n^2+1}$$

Divergence Test

$$\lim \frac{n^2+1}{3n^2+1} = \frac{1}{3} \neq 0$$

So series diverges

$$(24) \sum_{n=1}^{\infty} \frac{1}{n}$$

P-series

$p=1 \leq 1$ So series diverges

$$(25) \sum \frac{1}{n^2}$$

AND

$$(26) \sum \frac{1}{\sqrt{n}}$$

are both P-series

AND for you

$$(27) \sum \frac{n^2+1}{n^3+1}$$

LCT

known $\sum \frac{1}{n}$ diverges by p-series ($p=1 \leq 1$)

$$L = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{\frac{n^2+1}{n^3+1}} = \lim_{n \rightarrow \infty} \frac{n^3+1}{n^3+n} = 1$$

$0 < L < \infty$

So $\sum \frac{n^2+1}{n^3+1}$ diverges Since

$\sum \frac{1}{n}$ diverges

$$(28) \sum \frac{n^2+1}{n^4+1}$$

LCT to $\sum \frac{1}{n^2}$

which converges since $p=2 > 1$ by p-series

$$L = \lim_{n \rightarrow \infty} \frac{\frac{n^2+1}{n^4+1}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n^4+n^2}{n^4+1} = 1$$

Since $0 < 1 < \infty$ both series

converge.

NOTICE NUMERATOR

& DENOMINATOR

CAN SWITCH

(29) $\sum \frac{\sqrt{n+5}}{n^2+1}$ LCT to $\sum \frac{1}{n^{3/2}}$ which converges by p-Series Test since $p = \frac{3}{2} > 1$.

$$L = \lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n+5}}{n^2+1}}{\frac{1}{n^{3/2}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+5} \cdot n^{3/2}}{n^2+1} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4+5n^3}}{n^2+1} = 1$$

Since $0 < L = 1 < \infty$, we have both series converge.

$$\frac{\sqrt{n^6}}{n^2} = 1$$

(30) $\sum \frac{2^n + n^2}{3^n + n^3}$ LCT to $\sum \frac{2^n}{3^n}$ which converges by Geometric Series (#18)

$$L = \lim_{n \rightarrow \infty} \frac{\frac{2^n + n^2}{3^n + n^3} \cdot \frac{3^n}{2^n}}{\frac{2^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{6^n + n^2 \cdot 3^n}{6^n + n^3 \cdot 2^n}}{\frac{2^n}{3^n}} = 1$$

Since $0 < L = 1 < \infty$ both series converge.

(31) $\sum (-1)^n \frac{1}{n}$ AST $\lim \frac{1}{n} = 0$

So series converges

(32) $\sum (-1)^n \frac{1}{n^2}$ AST $\lim \frac{1}{n^2} = 0$

So series converges

Compare to
#24, #25
AND

#26

(33) $\sum (-1)^n \frac{1}{n^n}$ AST $\lim \frac{1}{n^n} = 0$

So series converges

$$(34) \sum_{n=1}^{\infty} \frac{n}{n!}$$

RATIO TEST

$$r = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{(n+1)!} \cdot \frac{n!}{n}}{\frac{n}{n!}} = \lim_{n \rightarrow \infty} \frac{n+1}{n} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$|r| = 0 < 1$ So series converges

$$(35) \sum_{n=1}^{\infty} \frac{e^n}{n!}$$

RATIO TEST

$$r = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}}{\frac{e^n}{n!}}$$

$$= \lim_{n \rightarrow \infty} \frac{e^{n+1}}{e^n} \cdot \frac{n!}{(n+1)!} = \lim_{n \rightarrow \infty} \frac{e^2 \cdot e^1}{e^n} \cdot \frac{1 \cdot (n-1)(n-2) \dots 3 \cdot 2 \cdot 1}{(n+1)(n)(n-1) \dots 3 \cdot 2 \cdot 1}$$

$$= \lim_{n \rightarrow \infty} \frac{e}{n} = 0$$

$r = 0 < 1$ So series converges

$$(36) \sum_{n=1}^{\infty} \frac{n^2}{e^n}$$

RATIO TEST

$$r = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2}}{\frac{n^2}{e^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{n^2} \cdot \frac{e^n}{e^{n+1}}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^2 \cdot \frac{e^n}{e^n \cdot e^1} = (1) \cdot \frac{1}{e} = \frac{1}{e}$$

$|r| = \frac{1}{e} < 1 \Rightarrow$ Series converges

(37.) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ Divergence test
 $\lim_{n \rightarrow \infty} \frac{e^n}{n^2} = \infty \neq 0$

So series diverges

(38.) $\sum \frac{n^n}{n!}$ Divergence Test
 $\lim \frac{n^n}{n!} = \infty \neq 0$

So series diverges

(39.) $\sum \frac{n!}{n^n}$ we did in class, Recall we got
 $r = \frac{1}{e} < 1$ series converges

(40.) $\sum \left(\frac{n^2+1}{3n^2+1} \right)^n$ Root Test
 $r = \lim \left(\left(\frac{n^2+1}{3n^2+1} \right)^n \right)^{1/n}$
 $= \lim \frac{n^2+1}{3n^2+1} = \frac{1}{3} < 1$ Series converges

41 & 42 Similar to 40

41 $\Rightarrow$ diverge with $r = \frac{3}{2}$ AND 42 converge with $r = \frac{2}{3}$

(43.) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ Root test
 $r = \lim (a_n)^{1/n} = \lim \left(\left(1 - \frac{1}{n}\right)^{n^2} \right)^{1/n} = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1}$

$|r| = e^{-1} < 1$ So series converges.

Note similarity with #43 AND #22.

ALSO NOTE THE DIFFERENCES.

$$44(a) \sum \frac{1}{n^2} x^n$$

$$\text{Ratio Test } r = \lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \left| \frac{\frac{1}{(n+1)^2} x^{n+1}}{\frac{1}{n^2} x^n} \right| = \lim_{n \rightarrow \infty} \frac{n^2}{(n+1)^2} |x|$$

$$r = |x| < 1 \quad \leftarrow \text{set } "< 1" \text{ \& solve for } x$$

$$|x| < 1$$

$$-1 < x < 1$$

Now test endpoints

$$x = 1$$

$$\sum \frac{1}{n^2} (1)^n = \sum \frac{1}{n^2}$$

P-series $p=2 > 1$ so series converges

$$x = -1$$

$$\sum \frac{1}{n^2} (-1)^n$$

$$\text{AST } \lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

so series converges

$$\text{IOC } -1 \leq x \leq 1$$

$$[-1, 1]$$

Radius is 1

$$(b) \sum \frac{1}{\sqrt{n}} x^n$$

RATIO TEST

$$r = \lim \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{1}{\sqrt{n+1}} x^{n+1}}{\frac{1}{\sqrt{n}} x^n} \right| = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+1}} |x|$$

$$r = |x| < 1 \quad \leftarrow \text{set } "< 1" \text{ \& solve for } x$$

$$= |x|$$

$$-1 < x < 1$$

Test endpoints $x = -1$ $x = 1$

$$x = -1$$

$$\sum \frac{1}{\sqrt{n}} (-1)^n$$

$$\text{AST } \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$$

so converge

$$x = 1$$

$$\sum \frac{1}{\sqrt{n}} (1)^n = \sum \frac{1}{\sqrt{n}}$$

P-series $p = \frac{1}{2} \leq 1$ diverge

$$-1 \leq x < 1$$

$$\text{IOC} = [-1, 1) \quad R = 1$$

44 c) $\sum_{n=1}^{\infty} \frac{1}{2^n} (x-1)^n$

RATIO TEST $r = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{1}{2^{n+1}} (x-1)^{n+1}}{\frac{1}{2^n} (x-1)^n} \right| = \lim_{n \rightarrow \infty} \frac{2^n}{2^{n+1}} |x-1| = \frac{1}{2} |x-1|$

$r = \frac{1}{2} |x-1| < 1$ (set < 1 & solve)

$\frac{1}{2} |x-1| < 1$

$|x-1| < 2$

$-2 < x-1 < 2$

$-1 < x < 3$

now test end points

$x = -1$

$x = 3$

$x = -1$ $\sum \frac{1}{2^n} (-1-1)^n$

$= \sum \frac{1}{2^n} (-2)^n = \sum (-1)^n$

divergence test

$\lim (-1)^n = \text{DNE}$

NOT zero

so series diverges

$x = 3$ $\sum \frac{1}{2^n} (3-1)^n = \sum \frac{1}{2^n} 2^n$

$= \sum 1$

Div. Test $\lim_{n \rightarrow \infty} 1 = 1 \neq 0$

so series diverges

$I \cup C = (-1, 3)$

$R = 2$

(44d) ANSWER ONLY

$I \cup C = [-1, 3]$

$R = 2$

(45 a) $f(x) = e^{3x}$ $a = 0$

$$f(x) = e^{3x}$$

$$f(0) = e^0 = 1$$

$$f'(x) = e^{3x} \cdot 3$$

$$f'(0) = 3$$

$$f''(x) = e^{3x} \cdot 3^2$$

$$f''(0) = 3^2$$

$$f'''(x) = e^{3x} \cdot 3^3$$

$$f'''(0) = 3^3$$

$$T(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \dots$$

$$= 1 + \frac{3}{1!} x + \frac{3^2}{2!} x^2 + \frac{3^3}{3!} x^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$$

(45 b) Answer only

$$\sin(x) = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \dots + \frac{(-1)^n}{(2n+1)!} x^{2n+1} + \dots$$

(45 c) $f(x) = \sin(x)$ centered at $a = \pi/2$

$$f(x) = \sin(x)$$

$$f(\pi/2) = \sin(\pi/2) = 1$$

$$f'(x) = \cos(x)$$

$$f'(\pi/2) = \cos(\pi/2) = 0$$

$$f''(x) = -\sin(x)$$

$$f''(\pi/2) = -1$$

$$f'''(x) = -\cos(x)$$

$$f'''(\pi/2) = 0$$

$$f^{(4)}(x) = \sin(x)$$

$$f^{(4)}(\pi/2) = 1$$

$$\begin{aligned} \text{So } T(x) &= f(\pi/2) + \frac{f'(\pi/2)}{1!} (x - \pi/2) + \frac{f''(\pi/2)}{2!} (x - \pi/2)^2 + \frac{f'''(\pi/2)}{3!} (x - \pi/2)^3 + \dots \\ &= 1 + 0 + \frac{-1}{2!} (x - \pi/2)^2 + 0 + \frac{1}{4!} (x - \pi/2)^4 + \dots \end{aligned}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x - \pi/2)^{2n}$$

(45d) $f(x) = \ln(x)$ center $x=1$

$$f(x) = \ln(x)$$

$$f(1) = \ln(1) = 0$$

$$f'(x) = \frac{1}{x} = x^{-1}$$

$$f'(1) = 1$$

$$f''(x) = -x^{-2}$$

$$f''(1) = -1$$

$$f'''(x) = 2x^{-3}$$

$$f'''(1) = 2$$

$$f^{(4)}(x) = -6x^{-4}$$

$$f^{(4)}(1) = -6$$

$$T(x) = f(1) + \frac{f'(1)}{1!}(x-1)^1 + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 + \dots$$

$$= 0 + \frac{1}{1!}(x-1)^1 + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4 + \dots$$

$$= (x-1) + \frac{-1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + \frac{-1}{4}(x-1)^4 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n$$

notice we start at $n=1$

46 a $f(x) = e^{3x}$

known $\rightarrow e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$

$$e^{3x} = 1 + (3x) + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots + \frac{(3x)^n}{n!} + \dots$$

$$= 1 + 3x + \frac{3^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \dots + \frac{3^n x^n}{n!} + \dots$$

(46b) $f(x) = \sin(x^2) - x^2$

known $\rightarrow \sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \dots$

$$\sin(x^2) = (x^2) - \frac{1}{3!}(x^2)^3 + \frac{1}{5!}(x^2)^5 - \frac{1}{7!}(x^2)^7 + \dots + \frac{(-1)^n}{(2n+1)!}(x^2)^{2n+1} + \dots$$

$$= x^2 - \frac{1}{3!}x^6 + \frac{1}{5!}x^{10} - \frac{1}{7!}x^{14} + \dots + \frac{(-1)^n}{(2n+1)!}x^{4n+2} + \dots$$

$$-x^2 \quad -x^2$$

$$\sin(x^2) - x^2 = -\frac{1}{3!}x^6 + \frac{1}{5!}x^{10} - \frac{1}{7!}x^{14} + \dots + \frac{(-1)^n}{(2n+1)!}x^{4n+2} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$$

$$(46c) \quad f(x) = \frac{\sin(x^2) - x^2}{x^6} \quad \text{at } a=0$$

an extension of the previous problem

$$\sin(x)$$

$$\sin(x^2)$$

all the same as 46b

$$\frac{\sin(x^2) - x^2}{x^6} = \frac{-\frac{1}{3!}x^6 + \frac{1}{5!}x^{10} - \frac{1}{7!}x^{14} + \dots + \frac{(-1)^n}{(2n+1)!}x^{4n+2} + \dots}{x^6}$$

$$\frac{\sin(x^2) - x^2}{x^6} = -\frac{1}{3!} + \frac{1}{5!}x^4 - \frac{1}{7!}x^8 + \dots + \frac{(-1)^n}{(2n+1)!}x^{4n-4} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n-4}$$

$$4n+2-6=4n-4$$

$$(46d) \quad f(x) = x e^{x^2} \quad \text{at } x=0$$

known

$$\rightarrow e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots + \frac{x^{2n}}{n!} + \dots$$

$$x e^{x^2} = x \left[1 + x^2 + \frac{1}{2!}x^4 + \frac{1}{3!}x^6 + \dots + \frac{1}{n!}x^{2n} + \dots \right]$$

$$= x + x^3 + \frac{1}{2!}x^5 + \frac{1}{3!}x^7 + \dots + \frac{1}{n!}x^{2n+1}$$