Math 6250: Test 1 Review

First you should be able to complete Quiz 1 and 2.

1 Foundations

1. State the Peano Axioms.
2. Use induction to prove something simple.
3. Define equivalence relation.
4. Be able to show a given relation is an equivalence relation.
5. Be able to show a given relation with an operation is well defined. Know what well defined means.
6. Define $(\mathbb{Z}, +, \cdot)$ using equivalence relation and show the operations are well defined.
7. Define $(\mathbb{Q}, +, \cdot)$ using equivalence relation and show the operations are well defined.

2 Functions and Cardinality

8. Know the definition of injective and surjective.
9. Prove a statement like: If f is injective and g is injective then $f \circ g$ is injective.
10. Show a given function is injective or surjective. For the folllowing determine if the function is injective or is not injective and prove. Also determine if the function is surjective or not and prove.
    (a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2x - 3$
    (b) $f : (1, 2) \to (-2, 4)$ where $f(x) = 2x - 3$
    (c) $f : (1, 2) \to (-1, 3)$ where $f(x) = 2x - 3$
    (d) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$
    (e) $f : (-\infty, 0) \to \mathbb{R}$ where $f(x) = x^2$
11. Prove $\mathbb{Q} \sim \mathbb{N}$, $\mathbb{Z} \sim \mathbb{N}$, $\mathbb{R} \not\sim \mathbb{N}$,

3 The Real and the Complex Numbers

12. State the definition of the Reals.

13. State the definition of a Field.

14. Compute a sup or inf.

15. Prove: Let $\alpha = \sup(A)$. If $\varepsilon > 0$ then there is some $x \in A$ so that $\alpha - \varepsilon < x \leq \alpha$.

16. Prove: Let $\alpha = \sup(A)$. If $\alpha \notin A$ then A is infinite.

17. Solve expressions like: $x^4 = 1$, $x^3 = 2$ and $x^4 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

4 Sequences

18. Show a given sequence is convergent.

    (a) $\lim_{n\to\infty} \frac{n+2}{n^2+4} = 0$
    (b) $\lim_{n\to\infty} \frac{n+2}{n+4} = 1$
    (c) $\lim_{n\to\infty} \frac{3n+2}{2n+\sqrt{n}+4} = \frac{3}{2}$
    (d) $\lim_{n\to\infty} \frac{n+2}{3n+\sin n+4} = \frac{1}{3}$

19. Prove if a sequence is convergent then it is bounded.

20. Know how to prove a property like: If (a_n) and (b_n) are convergent then $(a_n + b_n)$ is convergent. We learned four properties like this.

21. State the Monotone convergence Theorem.

22. Use the MCT to prove convergence for a recursively defined sequence.

5 Limits of Functions

23. Prove the following.

    (a) $\lim_{x\to -2} 3x - 1 = -7$
    (b) $\lim_{x\to -2} x^2 = 4$
    (c) $\lim_{x\to 4} x^2 + 2x$

(d) $\lim_{x\to 4} \frac{1}{x} = \frac{1}{4}$

(e) $\lim_{x\to 9} \sqrt{x} = 3$

24. Prove the following (use $\varepsilon - \delta$ definition).

(a) $f(x) = x^2$ is continuous at $x = -2$.

(b) $f(x) = x^2$ is continuous.

(c) $f(x) = \sqrt{x}$ is continuous at $x = 0$.

(d) $f(x) = \sqrt{x}$ is continuous at $x = 4$.

(e) $f(x) = \sqrt{x}$ is continuous.

(f) $f(x) = \frac{1}{x}$ is continuous at $x = 3$.

(g) $f(x) = \frac{1}{x}$ is continuous.