Test 1 PART 2

1. Use the Definition to compute $\int_1^3 (5x-4)\,dx$

$a=1$

$b=3$

$f(x)=5x-4$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$x_k = a + k\Delta x = 1 + k\left(\frac{2}{n}\right)$

$f(x_k) = f\left(1+\frac{2k}{n}\right) = 5\left(1+\frac{2k}{n}\right) - 4 = 5 + \frac{10k}{n} - 4 = 1 + \frac{10k}{n}$

$\int_1^3 5x-4\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x = \lim_{n\to\infty} \sum_{k=1}^{n} \left[1+\frac{10k}{n}\right]\cdot\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{2}{n} + \frac{20}{n^2}k\right] = \lim_{n\to\infty} \left[\frac{2}{n}\sum 1 + \frac{20}{n^2}\sum k\right]$

$= \lim_{n\to\infty} \frac{2}{n}(n) + \frac{20}{n^2}\cdot\frac{n(n+1)}{2} = 2 + 10 = 12$

You can check by using FTC.

$\int_1^3 (5x-4)\,dx = \frac{5x^2}{2} - 4x \Big|_1^3 = \frac{5(9)}{2} - \frac{5(1)^2}{2} - \left[4(3) - 4(1)\right]$

$= \frac{5}{2}(8) - 8 = 20 - 8 = \underline{\underline{12}}$

A good check but not for any points.

2. $a(t) = 6t - 12e^{2t}$

$V_0 = 4$

$S_0 = -3$

a) FIND $V(t)$ & $S(t)$

b) FIND V when $t = 3$.

(a) $a(t) = 6t - 12e^{2t}$

$v(t) = \int 6t - 12e^{2t}\,dt = 3t^2 - 6e^{2t} + C$

since $V_0 = 4$

$V(0) = 3(0)^2 - 6e^0 + C = 4$

$-6 + C = 4$

$C = 10$

$\boxed{V(t) = 3t^2 - 6e^{2t} + 10}$

$s(t) = \int v(t)\,dt = \int 3t^2 - 6e^{2t} + 10\,dt = t^3 - 3e^{2t} + 10t + C$

since $S_0 = -3$

$S(0) = (0)^3 - 3e^0 + 10(0) + C = -3$

$-3 + C = -3$

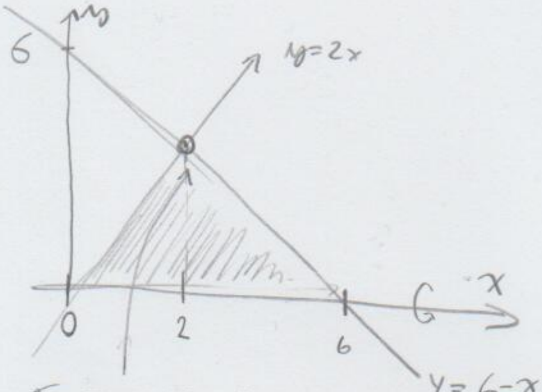
$C = 0$

$\boxed{s(t) = t^3 + -3e^{2t} + 10t}$

(b) $V(3) = 3(3)^2 - 6e^{2(3)} + 10$

$= 27 - 6e^6 + 10 = 37 - 6e^6$

3. $y = 2x$ $\quad y = 6 - x \quad$ x-axis [Revolve around the x-axis.]



[FIND Pt of intersection

$2x = 6 - x$ $\quad$ $y = 2x$

$3x = 6$ $\quad$ $y = 2(2) = 4$

$x = 2$ $\quad$ $(2,4)$]

[Use Disc's ← needs two regions!!]

$$V = \int_0^2 \pi (2x)^2 dx + \int_2^6 \pi (6-x)^2 dx$$

$$= \pi \int_0^2 4x^2 dx + \pi \int_2^6 [36 - 12x + x^2] dx$$

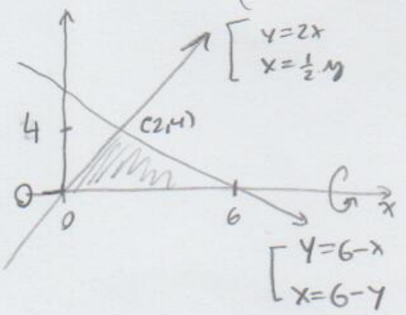
$$= \pi \left. \frac{4x^3}{3} \right|_0^2 + \pi \left[36x - \frac{12x^2}{2} + \frac{x^3}{3} \right]_2^6$$

$$= \pi \frac{4}{3}[2^3 - 0^3] + \pi \left[36[6-2] - 6[6^2 - 2^2] + \frac{1}{3}[6^3 - 2^3] \right]$$

$$= \pi \frac{32}{3} + \pi \left[144 - 192 + \frac{1}{3}(216 - 8) \right]$$

$$= \pi \left[-48 + \frac{32}{3} + \frac{208}{3} \right] = \pi \left[-48 + \frac{240}{3} \right] = \boxed{\pi 32}.$$

ANOTHER WAY (use Shells)



$$V = \int 2\pi y [R - L] dy$$

$$= \int_0^4 2\pi y \left[6 - y - \frac{1}{2} y \right] dy = 2\pi \int \left[6y - \frac{3}{2} y^2 \right] dy$$

$$= 2\pi \left[6\frac{y^2}{2} - \frac{1}{2} y^3 \right]_0^4 = 2\pi \left[3(16) - \frac{1}{2}(4)^3 \right] = 2\pi[16]$$

$$= \boxed{32\pi}$$