

MATH 2320 Practice Test 2

1 Techniques of integration

1. $\int \frac{1}{(4-9x^2)^{3/2}} dx$
2. $\int \frac{1}{(1+x^2)^{3/2}} dx$
3. $\int \frac{1}{(x^2-16)^{3/2}} dx$
4. $\int \frac{1}{x^2-x+6} dx$
5. $\int \frac{3x^2-x}{(x^2+1)(x-1)} dx$
6. $\int \frac{x^3+2x^2+x+1}{x^2(x^2+1)} dx$
7. $\int_1^\infty \frac{1}{x^2} dx$
8. $\int_1^\infty \frac{1}{\sqrt{x}} dx$
9. $\int_1^5 \frac{1}{x-1} dx$

2 Limits of sequences

10. Compute the following limits

- (a) $\lim_{n \rightarrow \infty} \frac{1}{n^2}$
- (b) $\lim_{n \rightarrow \infty} \frac{\sqrt{n}(3n^2-2n+1)}{5n^2+8}$
- (c) $\lim_{x \rightarrow 0} (1+5x^2)^{1/x}$
- (d) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$
- (e) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2}\right)^n$
- (f) $\lim_{n \rightarrow \infty} \frac{1}{n!}$
- (g) $\lim_{n \rightarrow \infty} \frac{1}{n^n}$
- (h) $\lim_{n \rightarrow \infty} \frac{n!}{(2n!)}$
- (i) $\lim_{n \rightarrow \infty} \frac{(n!)^2}{(2n!)}$
- (j) $\lim_{n \rightarrow \infty} \frac{e^n}{2^n}$

- (k) $\lim_{n \rightarrow \infty} \frac{\ln(n)}{n^2}$
 (l) $\lim_{n \rightarrow \infty} \frac{n^{23}+10}{e^n}$

3 Telescoping

- (a) $\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$
 (b) $\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$

4 Geometric Series

- (a) $\sum_{k=1}^{\infty} 3^{-k}$
 (b) $\sum_{k=1}^{\infty} \frac{1}{3^{-k}}$
 (c) $\sum_{k=17}^{\infty} 11 \cdot 3^{-k}$
 (d) $9 + -3 + 1 + -\frac{1}{3} + \frac{1}{9} + \dots$

5 Divergence Test

- (a) $\sum_{k=1}^{\infty} \frac{k^2+1}{3k^2+1}$
 (b) $\sum_{k=1}^{\infty} [1 - \frac{2}{k}]^k$

6 Integral Test

- (a) $\sum_{k=1}^{\infty} e^{-k}$

$$(b) \sum_{k=17}^{\infty} \frac{1}{k \ln(k)}$$

7 P-Series Test

$$(a) \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$$

$$(b) \sum_{k=1}^{\infty} \frac{k + k^2}{k^4}$$

8 Limit Comparison Test

$$(a) \sum_{k=1}^{\infty} \frac{5k^2}{\sqrt{4k^3 + 7}}$$

$$(b) \sum_{k=1}^{\infty} \frac{3k^2}{\sqrt{k^5 + 9}}$$

$$(c) \sum_{k=1}^{\infty} \frac{1}{3^k + k}$$

9 Alternating Series Test

$$(a) \sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$$

$$(b) \sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$$

$$(c) \sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2}$$

$$(d) \sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2 + 4}$$

10 Ratio Test

(a) $\sum_{k=1}^{\infty} \frac{k}{2^k}$

(b) $\sum_{k=1}^{\infty} \frac{1}{k!}$

(c) $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

(d) $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

(e) $\sum_{k=1}^{\infty} \frac{(k!)^2}{k^k}$

(f) $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

(g) $\sum_{k=1}^{\infty} \frac{(2k)!}{k^k}$

11 Root Test

(a) $\sum_{k=1}^{\infty} \left[\frac{1}{k}\right]^k$

(b) $\sum_{k=1}^{\infty} \left[\frac{3k^2 + 1}{5k^2 + 2k + 1}\right]^k$

(c) $\sum_{k=1}^{\infty} \left[1 - \frac{2}{k}\right]^{k^2}$

(d) $\sum_{k=1}^{\infty} \left[1 + \frac{2}{k}\right]^{k^2}$

12 Taylor Polynomials

- (a) Find the Taylor polynomial for $f(x) = e^x$ for $n = 3$ at $a = 0$. Estimate $e^{0.1}$.

- (b) Find the Taylor polynomial for $f(x) = \sin(2x)$ for $n = 3$ at $a = 0$.
Estimate $\sin(0.2)$.
- (c) Find the Taylor polynomial for $f(x) = \ln(x)$ for $n = 4$ at $a = 1$.
Estimate $\ln(1.01)$.