Please answer the questions showing your work completely and using correct grammar. You may use your notes your book or the online notes. Please do not search for questions on line or ask others for answers.

Name and date: ______________________________

1. Show $(0,1) \sim (6,9)$. Be certain to defie your funcation as well as show that it is 1-1 and onto.

2. Assume (a_n) converges to a. Prove that $a_n^2 \to a^2$. Use the $\varepsilon - N$ definition.

3. Find the limit and prove (use the $\varepsilon - N$ definition) your answer is the limit for the sequence $a_n = \frac{n+1}{2n+5}$.

4. Find the limit and prove (use the $\varepsilon - \delta$ definition) your answer is the limit for

$$\lim_{x\to 2} 2x^2 + 3$$

5. Prove the trigonometric identity

$$\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha).$$

6. Prove
   if f is differentiable at $x = c$ then f is continuous at $x = c$.

7. Use the MCT to prove convergence for a recursively defined sequence. Let $a_1 = 1$, and $a_{n+1} = \frac{a_n+1}{a_n+2}$ for all $n \in \mathbb{N}$. Make certain to state the MCT.

   Also what is the limit of this sequence? Do you notice anything else about this sequence?

8. Prove $f : [0,\infty) \to \mathbb{R}$ is continuous where $f(x) = \sqrt{x}$. Do this in two parts
   (a) Show f is continuous for all $c \in (0,\infty)$.
   (b) Show f is continuous at $x = 0$.

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the following property

$$\text{for all } x, y \in \mathbb{R} \text{ we have } |f(x) - f(y)| \le |x-y|^3.$$

   Using the $\varepsilon - \delta$ show that f is continuous.

10. We will find π.

(a) Compute the Taylor Series for arctan(x) with the following steps

i. Compute the series for $\frac{1}{1-x}$.

ii. Use your series for $\frac{1}{1-x}$ and a substitution of $x = -y^2$ to get a series for $\frac{1}{1+y^2}$.

iii. For your series for $\frac{1}{1+x^2}$ Integrate to get a series for arctan(x).

(b) Now use your series for arctan(x) to find a series for $4\arctan(1)$. What is $4\arctan(1)$? Sum the first 20 terms of your series. Is it close to what you expect?