

Math 3160 - Test 1

Name: _____

1. Solve the following system of linear equations using row reduction.

$$\begin{cases} x_1 & -2x_2 & & & +x_5 & = 0 \\ x_1 & +x_2 & -x_3 & +6x_4 & & = 3 \\ 2x_1 & +2x_2 & & & -x_5 & = 1 \end{cases}$$

2. Solve the following system of linear equations using row reduction.

$$\begin{cases} x & +2y & +3z & = 7 \\ 3x & & -z & = -5 \\ & y & +z & = 3 \end{cases}$$

3. Solve the following system of linear equations using row reduction.

$$\begin{cases} x & +2y & +3z & = 1 \\ 4x & +5y & +6z & = 1 \\ 7x & +8y & +9z & = 1 \end{cases}$$

4. Solve the following system of linear equations using $AX = B$ and using the inverse of A .

$$\begin{cases} x & +y & +3z & = 1 \\ & +y & +6z & = 1 \\ x & +y & +9z & = 1 \end{cases}$$

5. Solve the following system of linear equations using Cramer's Rule.

$$\begin{cases} x & +2y & +3z & = -1 \\ 2x & +y & +6z & = 0 \\ x & +y & & = 1 \end{cases}$$

6. Let $\mathbf{v} = (-1, 0, 4)$ and $\mathbf{w} = (-1, 2, 0)$ be vectors in $\mathbb{R}^3$. And consider the following points $P(1, 0, 0)$, $Q(3, 0, 2)$ and $R(-1, 4, 0)$.

- (a) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.
- (b) Find a unit vector in the same direction as $\mathbf{v}$.
- (c) Find the equation of a line containing the two points P and Q .

7. Let $\mathbf{v} = (-1, 0, 4)$ and $\mathbf{w} = (-1, 2, 0)$ be vectors in $\mathbb{R}^3$. And consider the following points $P(1, 0, 0)$, $Q(3, 0, 2)$ and $R(-1, 4, 0)$.

- (a) Find a single vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

- (b) Find the equation of a plane containing the three points P , Q and R .
- (c) What is the area of the triangle containing the points P , Q and R ?

8. Define the planes P_1 and P_2 as follows:

$$P_1 : x - 2y + z = -3$$

$$P_2 : x - 3y + z = 4$$

- (a) What are the two normal vectors for the above planes.
 - (b) The two planes above intersect in a line. Find the equation of that line.
9. Let $P_1(1, 0, 2, 0)$, $P_2(-1, 3, 0, 2)$, $P_3(0, 0, 0, 2)$ and $P_4(1, 1, 4, 0)$ be points in $\mathbb{R}^4$.
- (a) Find the angle between the vectors formed by $\vec{P_1P_2}$ and $\vec{P_1P_3}$.
 - (b) Find the equation of the line containing P_1 and P_4 .
 - (c) Find the equation of the plane (hyper-plane) containing P_1 , P_2 , P_3 and P_4 .