Math 3160 - Final Exam

Name:____

No calculators or any other electronic devices permitted. For credit you must justify your answer.

1. Solve the following system of linear equations.

$$\begin{cases} 2x_1 + 2x_2 + 3x_3 - 2x_5 = 12 \\ 2x_2 - x_3 + x_4 = 0 \\ -x_1 - x_3 = -6 \end{cases}$$

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2. The linear transformation T is given by the formula $T(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = \begin{bmatrix} x+y \\ y+z \\ x-z \end{bmatrix}$.

(a) Find the matrix, A, to represent the linear transformation T.

(b) Compute the basis for the Range of T, which is the Column Space of A.

(c) Find a basis for the null space of A, NULL(A).

(d) Compute the dimension of COL(A) and NULL(A). The dimension of the range of T is called the rank of T and the dimension of the null space is called the nullity.

(e) What is the dimension of the domain of T and the codomain of T? Again, compare Rank, Nullity and the dimension of the Domain. Do you see a relation?

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3. Let $\mathbf{v}_1 = (0, 0, 2, -1)$, $\mathbf{v}_2 = (1, 0, 0, -1)$, $\mathbf{v}_3 = (1, -1, -5, -1)$ and $\mathbf{v}_4 = (1, 1, 1, 1)$. Show the list of vectors is NOT linearally independent by finding a non-trivial linear combination of the vectors equal to zero.

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4. Let $\mathbf{v}_1 = (0,0,2,-1)$, $\mathbf{v}_2 = (1,0,0,-1)$, $\mathbf{v}_3 = (1,-1,-5,-1)$ and $\mathbf{v}_4 = (1,1,1,1)$. You have shown this collection of vector is not independent. Thus it is not a basis. Find a basis for the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

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5. Find all eigenvalues and corresponding eigenvectors .

$$A = \begin{bmatrix} 5 & 5 \\ 0 & 5 \end{bmatrix}$$

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6. Diagonalize.

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 0 \end{bmatrix}$$

7. For the E below find D (Hill cipher problem) and decipher the ciphertext message "XGCC". $E = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

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8. Let $\mathbf{v_1} = (1, 2, 3)$, $\mathbf{v_2} = (-1, 0, 0)$, $\mathbf{v_3} = (-2, 0, 1)$. Orthogonalize.

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9. Write down the definition of a linear transformation and prove the range of a linear transformation is a subspace. Hint use the two step subspace test.

Definition. Let V and W be vector spaces. A linear transformation $T : V \to W$ is **linear** if and only if

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Theorem. Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. then the range of T is a subspace.

proof.

$\blacksquare$

a	b	c	d	e	f	g	h	i	j	k	l	m
00	01	02	03	04	05	06	07	08	09	10	11	12

n	o	p	q	r	s	t	u	v	w	x	y	z
13	14	15	16	17	18	19	20	21	22	23	24	25

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\begin{aligned}
1 \cdot 1 \equiv 1 \equiv 1 \pmod{26}\\
3 \cdot 9 \equiv 27 \equiv 1 \pmod{26}\\
5 \cdot 21 \equiv 105 \equiv 1 \pmod{26}\\
7 \cdot 15 \equiv 105 \equiv 1 \pmod{26}\\
9 \cdot 3 \equiv 27 \equiv 1 \pmod{26}\\
11 \cdot 19 \equiv 209 \equiv 1 \pmod{26}\\
15 \cdot 7 \equiv 105 \equiv 1 \pmod{26}\\
17 \cdot 23 \equiv 391 \equiv 1 \pmod{26}\\
19 \cdot 11 \equiv 209 \equiv 1 \pmod{26}\\
21 \cdot 5 \equiv 105 \equiv 1 \pmod{26}\\
23 \cdot 17 \equiv 391 \equiv 1 \pmod{26}\\
25 \cdot 25 \equiv 625 \equiv 1 \pmod{26}
\end{aligned}
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