Math 3160 - Test 2.2

Name:____

1. Find standard equation for the plane (in $\mathbb{R}^3$) so that

   (a) the plane contains the point $P(2, 2, -1)$, $Q(1, 0, 3)$ and $R(0, -1, 0)$.

   (b) the plane contains the the point $P(2, 2, -1)$ and is parallel to the two vectors $(1, -2, 0)$ and $(1, 2, -4)$.

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2. Let $W = \{(x, y, z, w) \in \mathbb{R}^4 : \text{ where } x + z - w = 0\}$. Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

3. Let $S = \{(0,0,0,1), (1,1,1,1), (1,1,0,1)\}$.

   (a) Is S linearlly independent?

   (b) Is $(4,7,2,-1) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(4,7,2,-1)$?

   (c) Does S span $\mathbb{R}^4$?

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4. Let $B = \{(1,0),(0,1)\}$, $B_1 = \{(1,-1),(-1,2)\}$ and $B_2 = \{(1,1),(4,1)\}$.

   (a) Find the change of basis matrices for $P_{B\to B_1}$ and $P_{B_1\to B_2}$.

   (b) Find the coordinates of the point $(1,3)$ (given in the standard basis) relative to the bases B_1 and B_2.

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5. Write the matrix for the following transformations described below.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 30° counter-clockwise.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the x-axis is stretched by 3 and the y-axis is contracted be a factor of $1/2$.

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the x-axis is stretched by 3 and the y-axis is contracted be a factor of $1/2$ and then the plane is rotated by 30° counter-clockwise .

6. The linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is given by the formula

$$T(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = \begin{bmatrix} x + w \\ 2x + y + 3w \\ 3x + 2y + z + w \\ x + 2w \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear trasformation T.

(b) Compute the basis for the Range of T, which is the Column Space of A.

(c) Find a basis for the null space of A, NULL(A).

(d) Compute the dimension of COL(A) and NULL(A). The dimension of the range of T is called the rank of T and the dimension of the null space is called the nullity.

(e) What is the dimension of the domain of T and the codomain of T? Again, compare Rank, Nullity and the dimension of the Domain. Do you see a relation?

7. For the following matrices find the charactersitic equation, the eigenvalues and their cooresponding eigen vectors.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 5 & 0 & 4 \end{bmatrix}$$

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8. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a vector space V with the following properties:

- $\mathbf{v}_1$ is perpandicular to $\mathbf{v}_2$ and $\mathbf{v}_3$, and $\mathbf{v}_2$ is perpandicular to $\mathbf{v}_3$
- the norm of each vector is 1.

(a) What is $\mathbf{v}_2 \cdot \mathbf{v}_3$?

(b) What is the norm of $\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3$?

(c) What is the angle between $\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3$ and $\mathbf{v}_2$?

(d) What is the norm of $a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$ where $a, b, c \in \mathbb{R}$?
