Math 3160 - Test 1

Name:_____

No calculators and show all work.

1. Solve the following systems of linear equations using row reduction.

$$\begin{cases} x_1 - 2x_2 + 2x_3 - 6x_5 = 3 \\ 4x_4 = 6 \\ x_4 + x_5 = 0 \\ 2x_2 - 2x_3 + 6x_5 = 10 \end{cases}$$

2. Solve the following systems of linear equations using row reduction.

$$\begin{cases} x_1 & +2x_2 & +3x_3 & = 4 \\ x_1 & +x_2 & +x_3 & = 2 \\ -x_1 & & -x_3 & = 4 \end{cases}$$

3. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

$$\begin{cases} x_1 & +2x_2 & +4x_3 & = 0 \\ x_1 & & -x_3 & = 2 \\ x_1 & +x_2 & +x_3 & = -3 \end{cases}$$

4. Solve the following using Cramer's rules.

$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 0 \\ & -3x_2 & & = 2 \end{cases}$$

5. Find the parametric equations

   (a) for the line (in $\mathbb{R}^3$) so that the line contains the points $P(0, 3, 3)$ and $Q(-3, 4, 7)$.

   (b) for the plane (in $\mathbb{R}^3$) so that the plane contains the points $P(1, 3, 3)$, $Q(0, 3, 3)$ and $R(-3, 4, 7)$.

6. Row reduce the matrix B to REF and compute the determinant of the matrix B using the row reduction techniques.

$$B = \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 3 & 0 & -1 \\ 0 & 3 & 5 & -1 \\ 0 & 3 & 1 & 7 \end{bmatrix}$$

7. Let $P(1, -1, 4)$, $Q(0, 3, 4)$ and $R(2, 0, 0)$ be points in $\mathbb{R}^3$.

   (a) Compute the area of the triangle formed by the points P, Q and R.

   (b) What is the volume of the parallelpiped formed by the vectors $\vec{PQ}$, $\vec{PR}$ and $\hat{j}$?

   (c) What is the standard equation of the plane containing the triangle from Problem 7a?

   (d) What is the parametric (or vector) equation of the plane containing the triangle from Problem 7a?