

Please answer the questions showing your work completely and using correct grammar. No use of electronic devices allowed.

Name and date: _____

1. For the relation $\mathcal{R}$ defined on $\mathbb{Z}$ as follows

$$a\mathcal{R}b \iff 7|a - b.$$

prove $\mathcal{R}$ is an equivalence relation reflexive.

2. Prove the statement.

If f is surjective and g is surjective then $f \circ g$ is surjective.

3. Prove $(0, 4] \sim [3, 7)$.

4. Do one of the following:

- Prove: Let $\alpha = \sup(A)$. If $\varepsilon > 0$ then there is some $x \in A$ so that $\alpha - \varepsilon < x \leq \alpha$.
- State and prove the MCT.

5. Solve for all $x \in \mathbb{C}$ in: $x^6 = 1$.

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined

6. Find the following limit and prove your answer is correct using the $\varepsilon - N$ definition from class:

$$\lim_{n \rightarrow \infty} \frac{n^2 + 2}{3n^2 + 4n}$$

7. Prove: If (a_n) is convergent then (ka_n) is convergent where $k \in \mathbb{R}$. State the definition of convergence of a sequence.

8. Use the MCT to prove convergence for a recursively defined sequence. Let $a_1 = 8$, and $a_{n+1} = \sqrt{5a_n - 4}$ for all $n \in \mathbb{N}$. Make certain to state the MCT.