Math 6250 Presentation 1

For the following presentations in class write up your presentation completely.

Remember when you are presenting that you are presenting to a class of peers. If you aren't certain of a definition or statement it is likely that neither are your classmates. State these definitions. You are teaching the class. Make the presentation so that you could follow it if you are in the class. That is, be clear and thorough. Include these notes in your write-ups. I will distribute your write-ups to the class as well.

1. For the following series

$$S(n) = \sum_{k=1}^{n} 3k^2 - 3k + 1$$

   (a) Compute $S(1), S(2), S(3)$, and $S(4)$

   (b) Conjecture a formula for $S(n)$ for all $n \in \mathbb{N}$.

   (c) Prove using induction the formula you conjectures above is correct.

2. State (as we did in class) and Prove Pascal's Lemma. Here you do not need induction.

3. Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by

$$a\mathcal{R}b \iff 5|(a-b).$$

   (a) Note $3\mathcal{R}28$ since it is true that $5|(3-28)$ and 3 does not relate to 27 since it is false that $5|(3-27)$. Find three other pairs of integers that relate to each other.

   (b) State $\mathcal{R}$ is reflexive and prove it. That is, say

   For all $z \in \mathbb{Z}$ we have $z\mathcal{R}z$.

   Proof. $\cdots$ $\square$

   (c) State $\mathcal{R}$ is symmetric and prove it.

   (d) State $\mathcal{R}$ is transitive and prove it.

4. Let $\mathcal{R}$ be the relation on the set $\mathbb{N} \times \mathbb{N}$ defined by

$$(a_1, b_1)\mathcal{R}(a_2, b_2) \iff a_1 + b_2 = a_2 + b_1$$

   (a) Note the ordered pair $(1, 2)\mathcal{R}(8, 9)$ since $1 + 9 = 2 + 8$. Find four other ordered pairs of natural numbers that relate to each other.

   (b) State $\mathcal{R}$ is reflexive and prove it. That is, say

   For all pairs $(a, b) \in \mathbb{N} \times \mathbb{N}$ we have $(a, b)\mathcal{R}(a, b)$.

   Proof. $\cdots$ □

   (c) State $\mathcal{R}$ is symmetric and prove it.

   (d) State $\mathcal{R}$ is transitive and prove it.

5. Let $\mathcal{R}$ be the relation on the set $\mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ defined by

$$(a_1, b_1)\mathcal{R}(a_2, b_2) \iff a_1 b_2 = a_2 b_1$$

   (a) Note the ordered pair $(6, 4)\mathcal{R}(9, 6)$ since $6 \cdot 6 = 4 \cdot 9$. Find four other ordered pairs of integers that relate to each other.

   (b) State $\mathcal{R}$ is reflexive and prove it. That is, say

   For all pairs $(a, b) \in \mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ we have $(a, b)\mathcal{R}(a, b)$.

   Proof. $\cdots$ □

   (c) State $\mathcal{R}$ is symmetric and prove it.

   (d) State $\mathcal{R}$ is transitive and prove it.

6. This problem refers to Problem 3. Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by

$$a\mathcal{R}b \iff 5|(a - b).$$

   Is the usual operation of addition **well defined**? State what it means for addition to be well defined over this relation and prove that addition is well defined.