Name: ______________________________

1. Define the sequence a_n as follows

$$a_1 = 3 \text{ and for } n > 1 a_n = \sqrt{a_{n-1} + 2}$$

   (a) Compute the first four terms of the sequence.

   (b) Prove a_n is strictly decreasing. That is, prove $a_n > a_{n+1}$ for all $n \in \mathbb{N}$.

2. Prove. Let $a, b, c \in \mathbb{Z}$ with $a, c \neq 0$. If $a|b$ and $c|d$ then $ac|(ad + bc)$.

3. Find a pair of integers x, y so that $ax + by = gcd(a, b)$ for each pair below.

$$a = 133 \text{ and } b = 121$$

4. Let a, b, c be integers with $a \neq 0$. Prove if $a|bc$ and $\gcd(a, b) = 1$ then $a|c$.

5. Prove $\sqrt{5}$ is irrational.

6. For the following elements of S_4

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}, \gamma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

(a) Compute $\sigma \circ \tau$.

(b) Compute γ^3.

(c) Compute $(\tau \circ \sigma)^{-1}$.

7. Let
$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & 4 \end{pmatrix} \right\}.$$

(a) Write out the Cayley table.

(b) Show H is closed over $\circ$.

(c) What is G3?

(d) Show H over $\circ$ satisfies G3.

8. Define the following algebraic structure $(\mathbb{Z}, \boxtimes)$ by $a \boxtimes b = a + b - 3$.

   (a) What is the identity for $(\mathbb{Z}, \boxtimes)$?

   (b) What is the inverse for the element 3 in $(\mathbb{Z}, \boxtimes)$?