

Answer the questions completely and in complete English. No electronic devices are permitted.

Name: _____

1. Let $n \in \mathbb{Z}$. Prove If n is odd then $3n^2 + 5$ is divisible by 4.

2. Prove $2^n > n^2$ for all $n \in \mathbb{N}$ with $n > 5$.

3. Let $\mathcal{R}$ be a relation on $\mathbb{Z}$ be defined as follows

$$a\mathcal{R}b \iff 4|a + 3b.$$

Note that $\mathcal{R}$ is symmetric and transitive. Prove.

4. For the relations defined below what are the equivalence classes?

(a) $a\mathcal{R}b \iff 3|a-b$ on $\mathbb{Z}$

(b) $\mathcal{R} = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$ on $S = \{1,2,3\}$.

5. Prove If f is injective and g is injective then $g \circ f$ is injective.

6. Let $f : [0, 4] \rightarrow [2, 14]$ be defined by $f(x) = 3x + 2$. Prove f is a bijection.

7. What is the definition of cardinality, infinite, and finite.

8. Show $\mathbb{N} \sim T$ where $T = \{3n | n \in \mathbb{N}\}$.