Answer the questions completely and in complete English. No electronic devices are permitted.

Name: ______________________________

1. Let $n \in \mathbb{Z}$. Prove If n is odd then $n^2 - 1$ is divisible by 4.

2. Prove $2^n > n^2$ for all $n \in \mathbb{N}$ with $n > 5$.

3. Let R be a relation on $\mathbb{Z}$ be defined as follows

$$a\mathcal{R}b \iff 3|a+2b.$$

Note that $\mathcal{R}$ is symmetric and transitive. Prove.

4. For the relations defined below what are the equivalence classes?

   (a) $a\mathcal{R}b \iff 3|a+2b$ on $\mathbb{Z}$

   (b) $\mathcal{R} = \{(1,1),(2,2),(3,3),(1,2),(2,1))\}$ on $S = \{1,2,3\}$.

5. Prove If f is injective and g is injective then $g \circ f$ is injective.

6. Let $f : [0, 4] \to [-2, 2]$ be defined by $f(x) = -x + 2$.

   (a) Prove f is a bijection.

   (b) Find the inverse of f.

7. What is the definition of cardinality, infinite, and finite.

8. Show $\mathbb{N} \sim T$ where $T = \{2^n | n \in \mathbb{N}\}$