Name:_____

MA 3520: Quiz 3

Type all solutions. Your answers should be complete and in proper English.

(1) Which of the following are well ordered. No proof is needed.
   (a) $A = \mathbb{N}$
   (b) $B = \mathbb{Z}$
   (c) The set of all even integers.
   (d) $D = \{q \in \mathbb{Q} | q > 0\}$

(2) Prove. Let $B \subset A$. If A is a well ordered set of numbers than B is a well ordered set of numbers.

(3) Let $n \in \mathbb{N}$. Show $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

(4) Let $n \in \mathbb{Z}$ and $n > 4$. Show $n! > 2^n$.

(5) Let $n \in \mathbb{N}$. Show $4|5^n - 1$.

(6) Let R, a relation on $A = \{1, 2, 3\}$, be defined as follows

$$\mathcal{R} = \{(1, 2), (2, 3), (3, 1), (1, 1)\}$$

   (a) Compute $dom(\mathcal{R})$, $range(\mathcal{R})$, and $\mathcal{R}^{-1}$.
   (b) Is $\mathcal{R}$ reflexive symmetric or transitive? Show this.

(7) Let R be a relation on $\mathbb{Z}$ be defined as follows

$$a\mathcal{R}b \iff 4|a + 3b.$$

   (a) Write out seven element of this relation.
   (b) Note that $\mathcal{R}$ is reflexive, symmetric and transitive? Prove.