1 Joint Probability Distributions

1.1 Discrete

Let X, Y be two discrete random variables The function $p(x, y)$ is called a **Joint Probability Function** if

1. $0 \leq p(x, y) \leq 1$
2. $\sum_x \sum_y p(x, y) = 1$
3. $Pr[X = x, Y = y] = p(x, y)$

And the function

$$F(x, y) = Pr(X \leq x, Y \leq y)$$

is the **Joint Cumuluative Probability Function**.

The **Marginal probabilty** Functions are defined as
$p_X(x) = Pr(X = x) = \sum_y p(x, y)$ $p_Y(y) = Pr(Yy = x) = \sum_x p(x, y)$

Example 1.1.

	$p(x,y)$	y = *1*	*2*	*3*
	-1	*0.1*	*0.1*	*0.2*
x	*1*	*0.1*	*0.0*	*0.2*
	2	*0.1*	*0.1*	*0.1*

1. *What is the probabilty the* $x = 2$ *and* $y = 3$*?*
   $p(2, 3) = 0.1$

2. *What is the marginal probability of* x*?*
   Just sum across the row to get

   $$Pr(X = -1) = \sum_y p(-1, y) = p(-1, 1) + p(-1, 2) + p(-1, 3) == 0.1 + 0.1 + 0.2 = 0.4$$

   We can collect the three rows into a table the way that we normally do.

x	*-1*	*1*	*2*
$p_X(x)$	*0.4*	*0.3*	*0.3*

1.2 Continuous

Let X, Y be two continuous random variables The function $f(x, y)$ is called a **Joint Probability Density Function** if

1. $0 \leq f(x, y)$
2. $\int_{-infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dxdy = 1$

3. $Pr[a \leq X \leq b, c \leq Y \leq d] = \int_a^b \int_c^d f(x,y)dxdy$

Again the function

$$F(x,y) = Pr(X \leq x, Y \leq y) = \int_{-infty}^{x} \int_{-\infty}^{y} f(s,t)dsdt$$

is the **Joint Cumuluative Density Function**. Thus we get the

$$\frac{\partial^2}{\partial x \partial y} F(x,y) = f(x,y).$$

The **Marginal probabilty** Functions are defined as

- $f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$
- $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$

Example 1.2. *Let $f(x,y) = Cx^2y$ for $0 \leq x \leq 3$ and $0 \leq y \leq 4$ be the pdf for the random variables X and Y.*

1. *Find C.*

   I get $C = \frac{1}{72}$. So I can use $f(x,y) = \frac{1}{72}x^2y$.

2. *What is the probabilty that $x < 1$ and $y > 2$?*

   $Pr[x < 1, y > 2] = Pr[0 < x < 1, 2 < y < 4] = \int_0^1 \int_2^4 \frac{1}{72}x^2ydxdy = \frac{1}{36}$.

3. *What is the marginal probability of x?*
   Just integrate over all y-values

$$f_X(x) = \int_0^4 \frac{1}{72}x^2ydy = \frac{1}{9}x^2$$

1.3 Independence

We say X and Y are **independnt** if

$$f(x,y) = f_X(x)f_Y(y)$$

1.4 Conditional Probability Densities

- $f_{X|Y} = \dfrac{f(x,y)}{f_Y(y)}$
- $f_{X|Y} = \dfrac{f(x,y)}{f_X(x)}$

2 Order Statistics

$$Pr(Y_k \leq y) = \binom{n}{k} (F(y))^k (1 - F(y))^{n-k} + \binom{n}{k+1} (F(y))^{k+1} (1 - F(y))^{n-k-1} + \binom{n}{k+2} (F(y))^{k+2} (1 - F(y))^{n-k-2} + \cdots + \binom{n}{n} (F(y))^n$$