1 Know your Test 1 stuff

2 More Stuff

1. Given the following joint probability function below

	0	1
0	0.18	0.42
1	0.12	0.28

   (a) Find the marginal PF's of X and Y.

   (b) Determine if X and Y are independent

   (c) Find the conditional PF of Y given $X = 1$

   (d) Find the PF's of $Z = max(X, Y)$ and $W = min(X, Y)$

2. The joint PDF of X and Y is given by

$$f(x, y) = \frac{2}{(x+1)^2(y+1)^2} \text{ for } 0 < x < y < \infty$$

   and 0 everywhere else.

   (a) Find the marginal PDF's of X and Y. Also determine if X and Y are independent.

   (b) Find the conditional PDF of X given Y = y and the conditional PDF of Y given X = x

   (c) Find the PDF of $W = min(X, Y)$

3. The lifetime of a cat is given by X and the lifetime of a dog given by Y.

   - $f_X(x) = 0.05$, $0 < x < 20$
   - $f_Y(y) = 0.0625$, $0 < y < 16$
   - X and Y are independent.

   Calculate the probability that the cat will outlive the dog.

4. An insurance policy will pay 100% of a medical claim and 80% of a dental claim. Medical and dental claims are independent. Medical claims have pdf $f(x) = e^{-x}$ for $x > 0$ and dental claims have pdf $f(y) = 2e^{-2y}$ for $y > 0$. Find the probability that the total reimbursement is more than 2.

5. A company offers two insurance policies, a basic policy and a supplemental policy. If an employee purchases the supplemental policy she must first purchase the basic policy. Let X be the proportion of employees who purchased the basic policy and let Y be the proportion of employees who purchased the supplemental policy. Let

$$f(x, y) = C(x + y)$$

(a) What is the region of the xy-plane where f(x,y) is defined? Note X and Y are percentages and what relationship do X and Y have with each other?

(b) Find C.

(c) Given 10 % of the employees purchase the basic policy what is the probability that fewer than 5 % purchase the supplemental policy?

(d) Given that less than 10 % of the employees purchase the basic policy what is the probability that fewer than 5 % purchase the supplemental policy?

6. The lifetime (in months) of two components in some machine is given by the

$$f(x,y) = \frac{5}{125,000}(50 - x - y); 0 < x < 50 - y < 50$$

What is the probability that both components are still function in 20 months?

7. An insurance company insures a large number of drivers. Let X be the losses incurred under collision insurance and let Y be the losses incurred under the liability insurance where and Y have the joint pdf

$$f(x,y) = \frac{2x + 2 - y}{4}; 0 < x < 1, 0 < y < 2$$

What is the probability that the total loss is at least 1?

8. Compute the probability that $X < 0.2$ for the following pdf

$$f(x,y) = 6(1 - x - y); x > 0, y > 0, x + y < 1$$

9. Let X and Y be RV with joint pdf

$$f(x,y) = 15y; \text{ for } x^2 < y < x$$

(a) Find the marginal density for Y.

(b) Find the marginal density for X.

(c) Find COV(X,Y).

10. We asked 9 audience members of the movie Rampage (a Dwayne "the Rock" Johnson movie) to rank the film from 1 to 10. We consider the audience to be iid with pdf

$$f(x) = \frac{1}{333}x^2; \text{ for } 1 \leq x \leq 10$$

(a) What is the probability that a single audience member ranked the film as a 7 or higher?

(b) What is the probability the the second highest ranking was 7 or higher?

(c) What is the probability the the third highest ranking was 9 or higher?

11. We will randomly select a person and ask her to rank the movie Rampage and the movie Definitely Maybe (we will use a scale of 0 to 1). Assume the rankings have the following pdf

$$f(x,y) = C(1+x-y)^2; \text{ for } 0 \le x \le 1, 0 \le y \le 1,$$

(a) Without doing any mathematics do you expect these scores to be independent?

(b) Compute the marginal distributions for X and for Y.

(c) Are X and Y independent?

(d) Compute the COV(X,Y).

3 And More stuff

12. Let $X_1, \cdots, X_{20}$ be iid with mean 1 and variance 2. Calculate

- $E(X_1 + \cdots + X_{20})$
- $E(20X_1)$
- $VAR(X_1 + \cdots + X_{20})$
- $VAR(20X_1)$

13. The joint PF of X and Y is

	0	1
0	0.2	0.4
1	0.3	0.1

(a) Compute COV(X,Y)

(b) Compute the correlation coefficient given by

$$\rho = \frac{COV(X,Y)}{SD(X)SD(Y)}$$

14. The joint PF of X and Y is

$$f(x,y) = 6/7(1+x-y)^2; \text{ for } 0 \le x \le 1, 0 \le y \le 1$$

(a) Compute COV(X,Y). You did this in problem 11).

(b) Compute the correlation coefficient given by

$$\rho = \frac{COV(X,Y)}{SD(X)SD(Y)}$$

15. The joint PF of X and Y is

$$f(x,y) = xe^{-y};\ \text{for } 0 \le x \le 1, 0 \le y$$

(a) Compute M(s,t).

(b) Compute $\frac{\partial M(0,0)}{\partial s}$.

(c) Compute E(X).