1. Probability of rolling a 5 or 6 on a fair die is $\frac{1}{3}$

(a) $P_r(X=3) = \binom{20}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{17} = \frac{20!}{17!\,3!}\ \frac{2^{17}}{3^{20}}$

(b) $P_r(X>3) = 1 - P_r(X \leq 3)$

$= 1 - \left[P_r(X=0) + P_r(X=1) + \cdots + P_r(X=3)\right]$

$= 1 - \left[\binom{20}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{20} + \binom{20}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{19} + \cdots + \binom{20}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{17}\right]$

$= 1 - \left[\left(\frac{2}{3}\right)^{20} + 20\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{19} + \cdots + \frac{20!}{17!\,3!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{17}\right]$

(c) MEAN $\mu = pn$

$= \left(\frac{1}{3}\right)(20) = \frac{20}{3}$

(d) VARIANCE $\sigma^2 = npq = 20\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{40}{9}$

2. (a) FIND Z SCORE OF SAT SCORE OF 1020?

$$Z = \frac{X - \mu}{\sigma} = \frac{1020 - 900}{100} = \frac{120}{100} = 1.2$$

(b) FIND Z SCORE OF ACT SCORE OF 27?

$$Z = \frac{X - \mu}{\sigma} = \frac{27 - 21}{5} = \frac{6}{5} = 1.2$$

(b)
(c)
from chart as in class

③ $n = 1000$ $\qquad p = \frac{1}{3}$

(a) $\mu_X = np = 1000(\frac{1}{3}) = 333\frac{1}{3}$

$\sigma_X^2 = npq = 1000(\frac{2}{3})(\frac{1}{3}) = 222.2$

So $\sigma_X = \sqrt{222.2} = 14.9$

(b) So the BINOMIAL DISTRIBUTION with $n = 1000$, $p = \frac{1}{3}$

is approximately the Normal Distribution

with $\mu = 333.3$ AND $\sigma = 14.9$

By the CLT.

So $P_r(X \leq 350) \simeq P_r(N \leq 350)$

some books may like $P_r(N \leq 350.5)$. Why?

So Z-SCORE IS

$$Z = \frac{350.5 - 333.3}{14.9}$$

AND LOOK UP Z SCORE ON chart.

(c) $Z_{.90} = 1.28$ from chart

(d) $X_{.90} = Z_{.90}\,\mu + \sigma = (1.28)(333.3) + 14.9$

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$$f(x) = \frac{1}{7} e^{-x/7} \qquad x > 0$$

(a) $E(x)$

(b) $VAR(x)$

(c) $P_r(X \leq 6)$

$$\begin{array}{ccc} x & & \frac{1}{7}e^{-x/7} \\ 1 & + & -e^{-x/7} \\ 0 & - & 7e^{-x/7} \\ & + & \end{array}$$

(a) $E(x) = \int x\, f(x)\, dx = \int_0^\infty x \frac{1}{7} e^{-x/7}\, dx$

$$= \left[-x e^{-x/7} - 7e^{-x/7}\right]_{x=0}^{\infty}$$

$$= \overset{x=\infty}{[0 - 0]} - \overset{x=0}{[0 - 7e^0]} = \boxed{7}$$

$$\begin{array}{ccc} x^2 & & \frac{1}{7}e^{-x/7} \\ 2x & + & -e^{-x/7} \\ 2 & - & 7e^{-x/7} \\ 0 & + & -49e^{-x/7} \end{array}$$

(b) $E(x^2) = \int x^2 f(x)\, dx = \int_0^\infty x^2 \frac{1}{7} e^{-x/7}\, dx$

$$= \left[x^2(-e^{-x/7}) - 2x(7e^{-x/7}) + (2)(-49e^{-x/7})\right]_{x=0}^{\infty}$$

$$= \left[x^2(-e^{-x/7}) - 2x(7e^{-x/7}) + -98e^{-x/7}\right]_0^\infty$$

$$= 98 \cdot = 98$$

So $VAR(X) = E(x^2) - E(x)^2 = 98 - (7)^2 = 49$

So $STD(x) = \sqrt{49} = 7.$

(c) $P_r(X \leq 6) = \int_0^6 \frac{1}{7} e^{-x/7}\, dx = -e^{-x/7}\Big|_0^6 = \boxed{-e^{-6/7} + 1}$

5. $f(x) = \frac{1}{7} e^{-x/7} \qquad x > 0$

(a) $\Pr(X > 6) = \int_6^\infty f(x)\,dx = \int_6^\infty \frac{1}{7} e^{-x/7}\,dx$

$= -e^{-x/7}\Big|_6^\infty = -e^{-\infty/7} - \left(-e^{-6/7}\right)$

$= 0 + \frac{1}{e^{6/7}} = \boxed{e^{-6/7}}$

You can use Y_k ORDER STATISTICS OR THE BINOMIAL FROM PROB 1

(b) $\Pr(N \geq 16) = \binom{20}{16}\left(e^{-6/7}\right)^{16}\left(1-e^{-6/7}\right)^{4} + \binom{20}{17}\left(e^{-6/7}\right)^{17}\left(1-e^{-6/7}\right)^{3}$

$+ \cdots + \binom{20}{20}\left(e^{-6/7}\right)^{20}$

(c) Y_3 has died means Y_1, Y_2 AND Y_3 have died

so

$\Pr(Y_3 \leq 6) = \Pr(N \leq 17) = 1 - \Pr(N \geq 17)$

$= 1 - \left[\binom{20}{17}\left(e^{-6/7}\right)^{17}\left(1-e^{-6/7}\right)^{3} + \cdots + \binom{20}{20}\left(e^{-6/7}\right)^{20}\left(1-e^{-6/7}\right)^{0}\right]$

Probab. that light bulb is alive after 6 years