MA 2310 Test 3 Review Some Answers

1. So $\frac{dr}{dt} = 3$ in/sec AND $r = 7$ in.

We are looking for $\frac{dV}{dt}$.

We know $V = \frac{4}{3}\pi r^3$

So $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Thus $\frac{dV}{dt} = 4\pi(7)^2(3) = \boxed{588\pi \text{ in}^3/\text{sec}}$

(2) $\frac{dr}{dt} = 3$ AND $r = 6$ are given.

We want $\frac{dS}{dt}$? $S =$ SURFACE AREA

We know $S = 4\pi r^2$

So $\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(6)(3) = \boxed{144\pi \text{ in}^2/\text{sec}}$

surface area.

(3) We are given $\frac{dS}{dt} = 10$ in²/sec AND $r = 4$ in. We WANT $\frac{dV}{dt}$.

We will use both $V = \frac{4}{3}\pi r^3$ AND $S = 4\pi r^2$

FIRST $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$10 = 8\pi(4)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{10}{32\pi} = \frac{5}{16\pi} \text{ in/sec}$$

Next note we know $\frac{dS}{dt} = 10$, $r = 4$ AND $\frac{dr}{dt} = \frac{5}{16\pi}$.

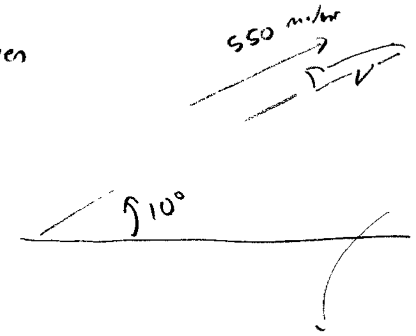
We will use $V = \frac{4}{3}\pi r^3$

So $$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi(4)^2\left(\frac{5}{16\pi}\right) = \boxed{20 \text{ in}^3/\text{sec}}$$

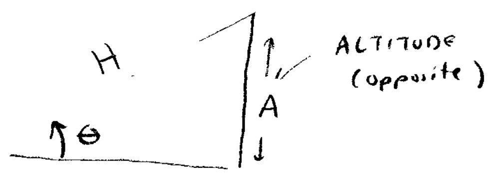
5. 3.11 (#20)

Given



WANT

HOW FAST IS ALTITUDE OF JET INCREASING



We WANT AN EQUATION THAT RELATES θ, Hypotenuse AND the altitude.

$$\sin\theta = \frac{A}{H}$$

$$A = \sin\theta \cdot H$$

plug in constants

H VARIABLE

A VARIABLE

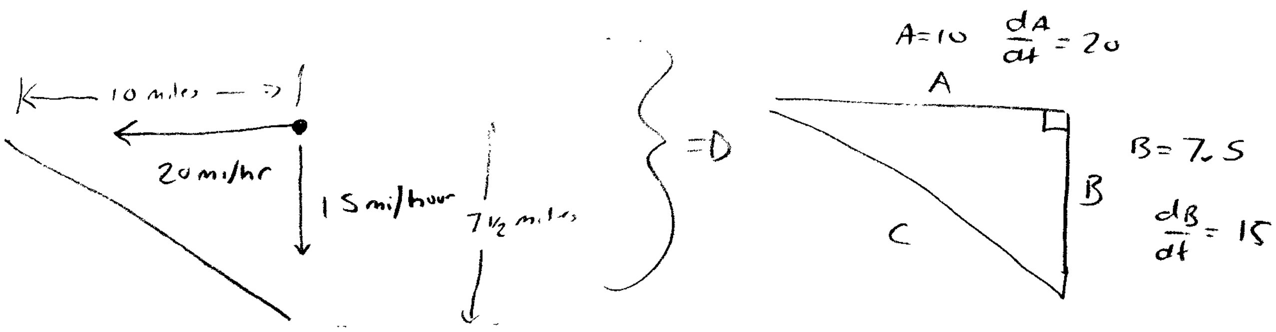
θ CONSTANT

$$A = \sin(10°)\, H$$

$$\frac{dA}{dt} = \sin(10°)\frac{dH}{dt}$$

$$\frac{dA}{dt} = \sin(10°)(550) \Rightarrow \boxed{\frac{dA}{dt} = 550 \cdot \sin(10°) \text{ mi/hour}}$$

5 § 3.11 ✱ 22



USE PYTHAGOREAN THEOREM

$$A^2 + B^2 = C^2$$

FIRST FIND C

$$10^2 + (7.5)^2 = C^2 \Rightarrow 100 + 56.25 = C^2$$

So $C = \sqrt{156.25} = 12.5$

Now take derivative $A^2 + B^2 = C^2$

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

NOTE A, B AND C ARE VARIABLE

plug in →

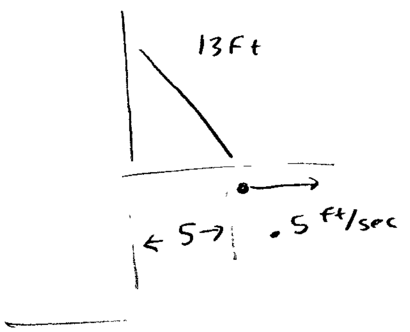
$$2(10)(20) + 2(7.5)(15) = 2(12.5)\left(\frac{dC}{dt}\right)$$

$$400 + 225 = 25\frac{dC}{dt}$$

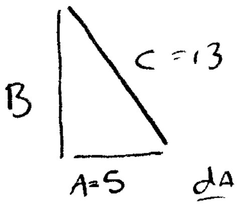
$$\frac{625}{25} = \frac{dC}{dt}$$

$$25 = \frac{dC}{dt} = \boxed{25 \text{ mi/hour}}$$

5, § 3.11 #23



$\Rightarrow$



$\frac{dA}{dt} = 0.5$

We WANT $\frac{dB}{dt}$

Pyth. Theorem

$A^2 + B^2 = C^2$ PLUG IN TO FIND B

$5^2 + B^2 = 13^2$

$B^2 = 169 - 25$

$\boxed{B = 12}$

$A^2 + B^2 = C^2$ plug in CONSTANTS

A, B VARIABLE

C CONSTANT

$A^2 + B^2 = 169$

$2A \frac{dA}{dt} + 2B \frac{dB}{dt} = 0$

plug in

$2(5)(0.5) + 2(12) \frac{dB}{dt} = 0 \Rightarrow \frac{dB}{dt} = -\frac{5}{24}$ ft/sec

Our answer is negative. Does this make sense?

(6 a) $f(x) = x^2 - x + 1$

$f'(x) = 2x - 1$

CP $\left\{ \begin{array}{l} 2x - 1 = 0 \\ \boxed{x = 1/2} \end{array} \right.$

$\rightarrow$ $- - -$ | $+ + +$ $f'(x)$

$x = 0$ test point $\quad$ $1/2$ $\quad$ $\boxed{x = 1}$

$f'(0) = 2(0) - 1$
$= -1$
$= -$

$f'(1) = 2(1) - 1$
$= 1$
$= +$

So $(1/2, 3/4)$ is a minimum

$f(1/2) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{1}{4} + \frac{1}{2} = 3/4$

(6 b) $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3} \cdot \frac{1}{x^{1/3}}$

FIND C.P. (1) $f'(x) = 0$ $\quad$ NO SOLUTION

(2) $f'(x)$ is UNDEFINED

DENOM $= 0$

$x^{1/3} = 0 \Rightarrow \boxed{x = 0}$

$- -$ | $+ + +$ $f'(x)$

$\boxed{x = -1 \text{ test}}$ $\quad$ 0 $\quad$ $\boxed{x = +1 \text{ test}}$

$f'(-1) = -$ $\qquad$ $f'(+1) = 1$

$(0, 0)$ is a minimum

$f(0) = 0^{2/3} = 0$

(69) $f(x) = x^4 - 2x^2 + 11$

$f'(x) = 4x^3 - 4x$

FIND C.P. $f'(x) = 0$

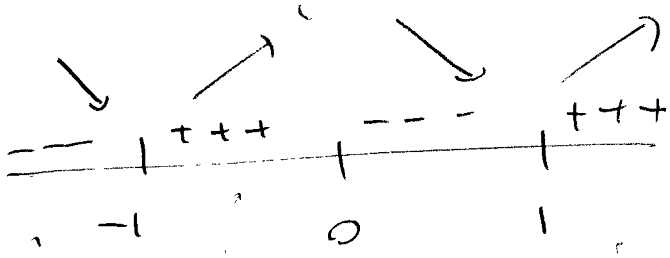
$4x^3 - 4x = 0$

$4x(x^2 - 1)$

$4x(x+1)(x-1) = 0$

$x = 0$ $\quad$ $x = -1$ $\quad$ $x = +1$

C.P.



test point $x = -2$ $\quad$ $x = -1/2$ $\quad$ $x = 1/2$ $\quad$ $x = 2$

$f'(-2) = 4(-2)(-2+1)(-2-1)$

$= (-)(-)(-)$

$= -$

$f'(-1/2) = 2(-1/2)(-\frac{1}{2}+1)(-\frac{1}{2}-1)$

$= (-)(+)(-)$

$= +$

AT $(-1, 10)$ AND $(+1, 10)$

f has minimums

AND AT $(0, 11)$ f has a maximum.

$f(0) = 11$

$f(-1) = (-1)^4 - 2(-1)^2 + 11$

$= 1 - 2 + 11 = 10$

$f(1) = 10$

(6 j) $f(x) = \dfrac{(x-2)^2}{(x+5)^3}$

$$f'(x) = \frac{2(x-2)(x+5)^3 - (x-2)^2\, 3(x+5)^2}{[(x+5)^3]^2}$$

$$= \frac{(x-2)(x+5)^2\,[2(x+5) - 3(x-2)]}{(x+5)^6}$$

$$= \frac{(x-2)(x+5)^2\,[-x+16]}{(x+5)^6} = \frac{(x-2)(-x+16)}{(x+5)^4}$$

FIND CP

(1) $f'(x) = 0$

$$\frac{(x-2)(-x+16)}{(x+5)^4} = 0$$

$$(x-2)(-x+16) = 0$$

$x = 2$ $\quad$ $\boxed{x = 16}$

(2) Set Denom = 0

$$(x+5)^4 = 0$$

$$\boxed{x = -5}$$

Is This a critical point? No! Why not? But still include on Numberline!

$x=-6$ test point		-5	$x=0$ test point		2		16	
$- - -$		$\mid$	$- - -$		$\mid$	$+ + + +$	$+$	$- - -$ $\to f'(x)$

AT $(2, 0)$ f has a minimum and

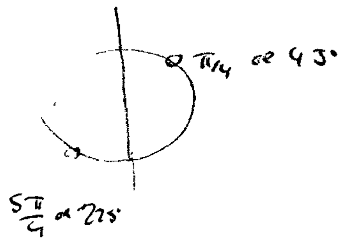
$f(2) = \dfrac{(2-2)^2}{(2+5)^3} = 0$

at $(16, \frac{4}{189})$ f has a maximum.

$f(16) = \dfrac{(16-2)^2}{(16+5)^3} = \dfrac{(14)^2}{(21)^3} = \dfrac{2^2 \cdot 7^2}{3^3 \cdot 7^3_{7}} = \dfrac{4}{27 \cdot 7} = \dfrac{4}{189}$

60) $f(x) = \sin(x) + \cos(x)$

$f'(x) = \cos(x) - \sin(x) = 0$

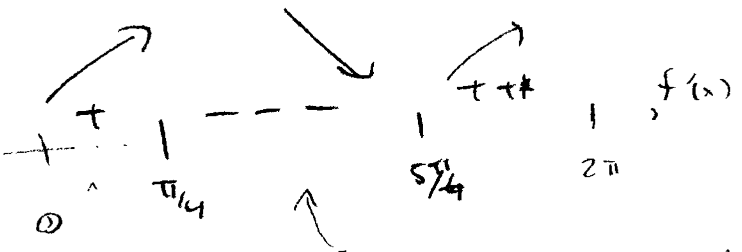


FIND C.P.

$\cos(x) = \sin(x)$

when $x = \pi/4$

or $5\pi/4$



$f'(\pi) = \cos(\pi) - \sin(\pi) = -1 - 0 = -1$

$f'(0) = \cos(0) - \sin(0) = 1$

So

at $(\pi/4, \sqrt{2})$ f attains a max and

$\left(f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}\right)$

at $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$ f attains a minimum.

(7a) $f(x) = x^2 - x + 1$

$f'(x) = 2x - 1$

$\boxed{\text{FIND C.P.}}$ $f'(x) = 0$

$2x - 1 = 0$

$x = 1/2$

$\boxed{\text{Test C.P.}}$

$f''(x) = 2$

So $f''(1/2) = 2 = +$ ∪

So

$(1/2, 3/4)$ is a minimum.

$f(1/2) = (\frac{1}{2})^2 - \frac{1}{2} + 1 = 3/4$

(7e) $f(x) = x^3 - 12x + 2$

$f'(x) = 3x^2 - 12$

FIND C.P.

$3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$3(x+2)(x-2) = 0$

$x = -2 \quad x = 2$

$\boxed{\text{Test CP.}}$

$f''(x) = 6x$

$\boxed{x = -2}$ $f''(-2) = -12 < 0$

∩ MAX

$\boxed{x = 2}$ $f''(2) = 12 > 0$ ∪

MIN

$(-2, 18)$ is a maximum for f

$f(-2) = -8 + 24 + 2 = 18$

$(2, -14)$ is a minimum for f

$f(2) = 2^3 + -12(2) + 2$

$= 8 - 24 + 2 = -14$

8.

(a) $f(x) = x^2 - x + 1$

$f'(x) = 2x - 1$

$f''(x) = 2$

FIND PPI $\quad f''(x) = 0$

$2 = 0$ } NO SOLUTION

SO NO INFLECTION points

8f. $f(x) = 2x^3 + 27x^2 + 20x$

$f'(x) = 6x^2 + 54x + 20$

$f''(x) = 12x + 54$

FIND PPI $\quad f''(x) = 0$

$12x + 54 = 0$

$12x = -54$

$x = -9/2 = -4\frac{1}{2}$

$- - - \quad | \quad + + + \quad f''(x)$

$-\frac{9}{2}$

test point $x = -5$ $\quad$ test point $x = 0$

$f''(-5) = 12(-5) + 54$

$= -60 + 54$

$= -$

$f''(0) = 54 = +$

$\left(-\frac{9}{2}, *\right)$ is a point of inflection

$f\left(-\frac{9}{2}\right) = 2\left(-\frac{9}{2}\right)^3 + 27\left(-\frac{9}{2}\right)^2 + 20\left(-\frac{9}{2}\right)$

10. (a) $\lim_{x\to 0} \frac{e^x-1}{x} \overset{(LH)}{=} \lim_{x\to 0} \frac{e^x}{1} = e^0 = \boxed{1}$

$\searrow \frac{e^0-1}{0} = \frac{1-1}{0} = \frac{0}{0}$ ✓

(b) $\lim_{x\to 0} \frac{e^{x^2}-1}{x} \overset{(LH)}{=} \lim_{x\to 0} \frac{2x\, e^{x^2}}{1} = 2(0)\, e^0 = 0$

(c) $\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} = \qquad = \boxed{}$

(e) $\lim_{x\to 0} \frac{e^x-1-x}{x^2} \overset{(LH)}{=} \lim \frac{e^x-1}{2x} \overset{(LH)}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$

$\searrow \frac{0}{0}$ $\qquad \searrow \frac{0}{0}$

(i) $\lim_{x\to\infty} \frac{\ln(x)}{3x^4-9} \overset{(LH)}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{12x^3} = \lim_{x\to\infty} \frac{1}{12x^4} = 0$

$\uparrow \frac{\infty}{\infty}$ $\qquad \searrow \frac{1}{\infty}$

(m) $\lim_{x\to 0^+} \frac{\ln(1+x)}{\frac{1}{x}} \overset{(LH)}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{-x^{-2}} = \lim_{x\to 0^+} \frac{1}{1+x} \cdot \frac{(-x^2)}{1} = \frac{-0}{1+0} = \boxed{0}$

$\uparrow \frac{0}{0}$