Test 2 Review Some Answers

1 a) $f(x) = \dfrac{x^2+1}{x^2} = \dfrac{x^2}{x^2} + \dfrac{1}{x^2}$

$= 1 + x^{-2}$

$f'(x) = 0 + -2x^{-3}$

(b) $f(x) = \dfrac{x^2}{x^2+1}$ ← use quotient rule

$\dfrac{N'D - ND'}{D^2}$

$f'(x) = \dfrac{2x \cdot (x^2+1) - x^2(2x+0)}{(x^2+1)^2}$

(c) $f(x) = 3e^{x^2}$

$f'(x) = 3e^{x^2} \cdot 2x$

(d) $f(x) = 3e^{x^2}\cot(x)$ PRODUCT RULE

$f'(x) = 3e^{x^2} \cdot 2x \cot(x) - 3e^{x^2} \cdot \cot(x)\csc(x)$

(e) $f(x) = \dfrac{1}{1+x^2} = (1+x^2)^{-1}$

so $f'(x) = -(1+x^2)^{-2}(2x) = \dfrac{-2x}{(1+x^2)^2}$

(1 f) $f(x) = \dfrac{e^x}{1+e^{2x}}$

QUOTIENT RULE

$$f'(x) = \frac{e^x(1+e^{2x}) - e^x(0+2e^{2x})}{(1+e^{2x})^2}$$

$$= \frac{e^x + e^{3x} - 2e^{3x}}{(1+e^{2x})^2} = \frac{e^x - e^{3x}}{(1+e^{2x})^2}$$

(1 j) $f(x) = \sec(\csc(x^3))$

$$f'(x) = -\sec(\csc(x^3))\tan(\csc(x^3)) \cdot \csc(x^3)\cot(x^3) \cdot 3x^2$$

(2. a) $x^3 - y^3 = 1$

$$3x^2 - 3y^2 \cdot y' = 0$$

$$\frac{3y^2 y'}{3y^2} = \frac{3x^2}{3y^2}$$

$$\boxed{y' = \frac{x^2}{y^2}}$$

(2b) $y^2 + \cos(y) = x$

$$2y \cdot y' + -\sin(y) \cdot y' = 1$$

$$\frac{y'[2y - \sin(y)]}{[2y - \sin(y)]} = \frac{1}{[2y - \sin(y)]}$$

$$y' = \frac{1}{2y - \sin(y)}$$

(2c) $x^3 y^3 = 1$

$$3x^2 y^3 + x^3 3y^2 \cdot y' = 0$$

$$\frac{x^3 3y^2 y'}{x^3 3y^2} = \frac{-3x^2 y^3}{x^3 \cdot 3y^2}$$

$$y' = -\frac{y}{x}$$

(2d) $3x^2 \sec(y) - y = x^3$

$$6x \cdot \sec(y) + 3x^2 \sec(y)\csc(y) \cdot y' - y' = 3x^2$$

$$3x^2 \sec(y)\csc(y) \cdot y' - y' = 3x^2 - 6x \sec(y)$$

I skipped a step. See if you can fill it in.

$$y' = \frac{3x^2 - 6x \sec(y)}{3x^2 \sec(y)\csc(y) - 1}$$

3

(2e) $\boxed{\sin(x^3y^3) = x^3+y^3}$

$$\cos(x^3y^3)\left[3x^2y^3 + x^3\cdot 3y^2y'\right] = 3x^2 + 3y^2\cdot y'$$

$$\cos(x^3y^3)\cdot 3x^2y^3 + \underline{\cos(x^3y^3)\cdot x^3 3y^2 y'} = 3x^2 + \underline{3y^2\cdot y'}$$

$$\cos(x^3y^3)\,3x^3y^2y' - 3y^2y' = 3x^2 - \cos(x^3y^3)\,3x^2y^3$$

$$\left[\cos(x^3y^3)3x^3y^2 - 3y^2\right]y' = 3x^2 - \cos(x^3y^3)(3x^2y^3)$$

$$y' = \frac{3x^2 - \cos(x^3y^3)\,(3x^2y^3)}{\cos(x^3y^3)\,3x^3y^2 - 3y^2}$$

(3a) $\boxed{y = x^x}$

$$\ln y = \ln x^x$$

$$\ln y = x\ln(x)$$

$$\frac{1}{y}\cdot y' = 1\ln(x) + x\cdot\frac{1}{x}$$

$$y' = y\left[\ln(x) + 1\right]$$

$$y' = x^x\left[\ln(x)+1\right].$$

(3b) $y = x^{\cos(x)}$

$\ln y = \ln x^{\cos(x)}$

$\ln y = \cos(x) \cdot \ln(x)$

$\frac{1}{y} y' = -\sin(x) \cdot \ln(x) + \cos(x) \cdot \frac{1}{x}$

$y' = y \left[-\sin(x) \cdot \ln(x) + \frac{\cos(x)}{x} \right]$

$y' = x^{\cos(x)} \cdot \left[-\sin(x) \ln(x) + \frac{\cos(x)}{x} \right]$

(3d) $y = (\cos(x) + x^2)^{x^3 + \sin(x)}$

$\ln y = \ln (\cos(x) + x^2)^{x^3 + \sin(x)}$

$\frac{d}{dx}$ $\ln y = [x^3 + \sin(x)] \cdot \ln(\cos(x) + x^2)$

$\frac{1}{y} y' = [3x^2 + \cos(x)] \cdot \ln(\cos(x) + x^2) + [x^3 + \sin(x)] \frac{1}{\cos(x) + x^2} (-\sin(x) + 2x)$

$y' = y \left[[3x^2 + \cos(x)] \ln(\cos(x) + x^2) + [x^3 + \sin(x)] \cdot \frac{-\sin(x) + 2x}{\cos(x) + x^2} \right]$

$y' = (\cos(x) + x^2)^{(x^3 + \sin(x))} [\quad — // — \quad]$

(4) (d) $f(x) = \dfrac{1}{1+x^2}$ at $x = -1$

POINT $(x_0, y_0) = (-1, ?)$

$y_0 = f(-1) = \dfrac{1}{1+(-1)^2} = \dfrac{1}{2}$ $\Rightarrow (-1, \frac{1}{2})$

SLOPE $f(x) = (1+x^2)^{-1}$

$f'(x) = -(1+x^2)^{-2}(2x)$

$f'(-1) = -($

(4e) Let $f(x) = x^{2x}$ FIND TANGENT LINE AT $x=1$.

POINT $(x_0, y_0) = (1, ?)$

$Y_0 = f(1) = 1^{2(1)} = 1^2 = 1$ $\Longrightarrow (x_0, y_0) = (1, 1)$

SLOPE

$f'(x) = ?$

$Y = x^{2x}$

$\ln y = \ln x^{(2x)}$

$\ln y = 2x \ln(x)$

$\frac{1}{y} y' = 2 \ln(x) + 2x \cdot \frac{1}{x}$

$y' = y[2\ln(x) + 2]$

$y' = x^{2x}[2\ln(x) + 2]$

So

$f'(x) = x^{2x}[2\ln(x) + 2]$

$m = f'(1) = 1^{2(1)}[2\ln(1) + 2] = 2$ $(\ln(1) = 0)$

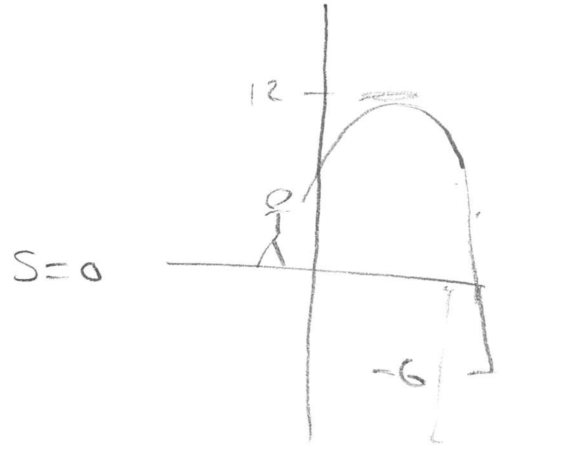
$y - y_0 = m(x - x_0)$

$\boxed{y - 1 = 2(x - 1)}$

$S(t)$

↑

displacement



Velocity

$V(t) = s'(t)$

accel.

$a(t) = s''(t)$

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5. $s(t) = -4.9t^2 + 98$

(a) FIND $V(t)$, $a(t)$.

$V = -9.8t$ $\quad$ $a = -9.8$

(b) When is displacement zero?

↑

want find t

$$S = 0$$

$$-4.9t^2 + 98 = 0$$

$$\frac{4.9t^2}{4.9} = \frac{98}{4.9}$$

$$t^2 = 20$$

$$t = \pm\sqrt{20}$$

$t = \sqrt{20}$ $= 2\sqrt{5}$ or $\cancel{t = -\sqrt{20}}$ ← ignore negative answers

$\boxed{5(c)}$ When is velocity zero?

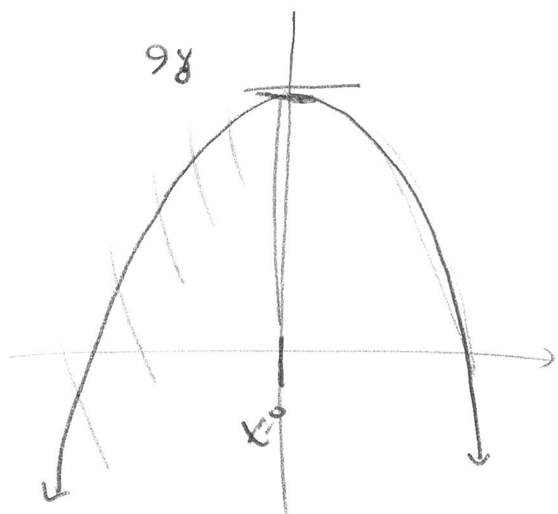
want t

$$V = 0$$

$$-9.8t = 0$$

$$\boxed{t=0}$$

(5d) Find max height?



Max height is when velocity is zero from part c)

we have $t=0$.

So height is

$$s(0) = -4.9(0)^2 + 98$$
$$= 98$$

(6) $s(t) = -t^3 - 2t^2 + 2t$

(a) FIND $a(t)$ & $v(t)$.

$$V(t) = s'(t) = -3t^2 - 4t + 2$$

$$a(t) = s''(t) = -6t - 4$$

(b) When does particle have position zero?

$$s(t) = 0$$

$$-t^3 - 2t^2 + 2t = 0$$

$$-t(t^2 + 2t + -2) = 0$$

$$t = 0 \qquad t^2 + 2t - 2 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

$t = 0 \qquad t = -1 + \sqrt{3} \qquad t = -1 - \sqrt{3}$

X Negative No good

PARTICLE HAS POSITION ZERO AT

$t = 0$ & $t = -1 + \sqrt{3}$.

6c) When does particle have velocity zero?

$v(t) = 0$

$-3t^2 - 4t + 2 = 0$

$t = \dfrac{-(-4) \pm \sqrt{16 - 4(-3)(2)}}{-6}$

$= \dfrac{4 \pm \sqrt{40}}{-6}$

$\cancel{t = \dfrac{4 + \sqrt{40}}{-6}}$ $\boxed{t = \dfrac{4 - \sqrt{40}}{-6}}$

NO ANSWER

Neg

6d) $v(1) = -3(1)^2 - 4(1) + 2 = \underline{\underline{-5}}$

9

(a) $f(x) = \arcsin(x)$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

(b) $f(x) = \arcsin(x^2)$

chain rule

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot (2x) = \frac{2x}{\sqrt{1-x^4}}$$

(c) $f(x) = \sec^{-1}(x^2)$

chain rule

$$f'(x) = \frac{1}{|x^2|\sqrt{1-(x^2)^2}} \cdot 2x$$

(d) $f(x) = x^2 \tan^{-1}(x)$

$$f'(x) = 2x \tan^{-1}(x) + x^2 \frac{1}{1+x^2}$$

(g) $f(x) = \cos(\tan(x^2))$

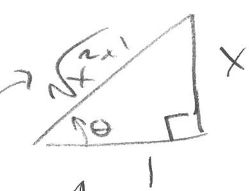
$$f'(x) = \sin(\tan(x^2)) \cdot \sec^2(x^2) \cdot 2x$$

9(h) $f(x) = \cos(\tan^{-1}(x))$

$$\cos(\tan^{-1}(x)) = \cos(\theta)$$

$$\theta = \tan^{-1}(x)$$

so $\tan(\theta) = x \implies \tan(\theta) = \frac{x}{1}$ $\quad \frac{\text{opp}}{\text{adj}}$



to FIND C

$$A^2 + B^2 = C^2$$

$$x^2 + 1^2 = C^2$$

$$\sqrt{x^2+1} = C$$

$$\cos(\theta) = \frac{1}{\sqrt{1+x^2}} \quad \frac{\text{adj}}{\text{hyp}}$$

$$f(x) = \cos(\tan^{-1}(x))$$

$$= \cos(\theta) = \frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-1/2}$$

$$\boxed{f'(x) = -\frac{1}{2}(1+x^2)^{-3/2} \cdot 2x}$$