

Show all work. No calculators allowed.
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Name and section: _____

1. Compute the following derivatives.

(a) $f(x) = \frac{e^x}{1+e^{2x}}$

(b) $f(x) = \tan(x^6 + 1)$

(c) $f(x) = e^x \cos(x^2)$

(d) $f(x) = \ln(x^2 + 1) \csc(x^3)$

(e) $f(x) = \cot(x^3 + 1)$

(f) $f(x) = (x - 1)^3(2x + 1)^5$

2. Find $\frac{dy}{dx}$ using implicit differentiation.

(a) $x^2\sqrt{y} = y^2 + x$

(b) $\sin(x^2 + y^3) = x^3 - 4$

3. Find $\frac{dy}{dx}$ using logarithmic differentiation technique.

(a) $y = x^{x^2+1}$

(b) $y = 2^x$

4. Find the equation of the tangent line for the following functions at the given point

(a) $f(x) = x^3$ at $x = 1$

(b) $f(x) = \cos(3x + 1)$ at $x = -1/3$

5. Let $s(t) = -5t^2 + 40t$ represent the height of a ball thrown upward.
- (a) Find the velocity and acceleration functions.
 - (b) When does the ball have velocity zero?
 - (c) What is the maximum height of the ball (for $t \geq 0$)?

6. Compute the derivatives of the inverse trigonometric functions

(a) $f(x) = \arcsin(x^3)$

(b) $f(x) = e^{3x+1} \tan^{-1}(x^2)$

7. Let $f(x) = -x^4 + 18x^2 + 98$.

- (a) Find $f'(x)$, $f''(x)$ and $f'''(x)$.
- (b) What is the slope of any horizontal line?
- (c) Find all x -values so that the tangent line to $f(x)$ is horizontal.

Extra Credit.

There are many myths and legends surrounding George Washington. We have heard myths about cherry trees, about horse riding and about the Natural Bridge in Virginia. It is said that Old George threw a rock from the base to the top of the Natural Bridge in Virginia. Look at Question 5. This question models George's rock toss in meters with time in seconds. A modern baseball pitcher can throw a ball about 40 meters per second. And the natural bridge is about 66 meters from base to the top.

Using Question 5

- What is speed George threw the rock at time $t = 0$? Does this sound reasonable to you?
- Did George throw the rock all the way to the top of the Natural Bridge?
- In your opinion is this legend plausible?