1 Related Rates

1. The radius of a spherical balloon is increasing at a rate of 3 inches per second. When the radius is 7 inches, how fast is the volume of the balloon increasing?
2. The radius of a spherical balloon is increasing at a rate of 3 inches per second. When the radius is 6 inches, how fast is the surface area of the balloon increasing?
3. The surface area of a spherical balloon is increasing at a rate of 10 square inches per second. When the radius is 4 inches, how fast is the volume of the balloon increasing?
4. The volume of a spherical balloon is increasing at a rate of 10 cubic inches per second. When the radius is 9 inches, how fast is the surface area of the balloon increasing?
5. See section 3.11 problems 20, 22, 23 and 24

2 Graphing

6. For the following functions find and classify all extremma using the first derivative test. Show all work including the first derivative number line and the (x, y) coordinates of the critical points.
   (a) $f(x) = x^2 - x + 1$
   (b) $f(x) = x^{2/3}$
   (c) $f(x) = x^{1/5}$
   (d) $f(x) = x^{-1/5}$
   (e) $f(x) = x^3 - 12x + 2$
   (f) $f(x) = 2x^3 + 27x^2 + 120x$
   (g) $f(x) = x^4 - 2x^2 + 11$
   (h) $f(x) = (x-2)^5(x-3)^6$
   (i) $f(x) = (x-2)^3(x-3)^3$
   (j) $f(x) = \frac{(x-2)^2}{(x+5)^3}$
   (k) $f(x) = xe^x$
   (l) $f(x) = e^{x^2}$
   (m) $f(x) = xe^{x^2}$
   (n) $f(x) = \sin(x) - x$ for $x \in [0, 2\pi]$
   (o) $f(x) = \sin(x) + \cos(x)$ for $x \in [0, 2\pi]$
   (p) $f(x) = x\cos(x) + \sin(x) - x^2$ for $x \in [0, 2\pi]$

7. For the following functions find and classify all extremma using the second derivative test. Show all work include the (x, y) coordinates of the critical points.

   (a) $f(x) = x^2 - x + 1$
   (b) $f(x) = x^{2/3}$
   (c) $f(x) = x^{1/5}$
   (d) $f(x) = x^{-1/5}$
   (e) $f(x) = x^3 - 12x + 2$
   (f) $f(x) = 2x^3 + 27x^2 + 120x$
   (g) $f(x) = x^4 - 2x^2 + 11$
   (h) $f(x) = x^5 - x^3$
   (i) $f(x) = x^6 - x^4$
   (j) $f(x) = e^{x^2}$

8. For the following functions find and classify all possible points of inflection. Show all work including the second derivative number line and the (x, y) coordinates of the points of inflection.

   (a) $f(x) = x^2 - x + 1$
   (b) $f(x) = x^{2/3}$
   (c) $f(x) = x^{1/5}$
   (d) $f(x) = x^{-1/5}$
   (e) $f(x) = x^3 - 12x + 2$
   (f) $f(x) = 2x^3 + 27x^2 + 120x$
   (g) $f(x) = x^4 - 2x^2 + 11$
   (h) $f(x) = x^5 - x^3$
   (i) $f(x) = x^6 - x^4$
   (j) $f(x) = e^{x^2}$

3 Optimization

9. Could be a chicken coop problem or a box optimization like in class. Also try section 4.4 problems 14, 15, 16 and 17.

4 L'Hopital's Rule

10. Find the indicated limit.

    (a) $\lim_{x\to 0} \frac{e^x-1}{x}$

    (b) $\lim_{x\to 0} \frac{e^{x^2}-1}{x}$

    (c) $\lim_{x\to 0} \frac{e^{x^2}-1}{x^2}$

    (d) $\lim_{x\to 0} \frac{e^{x^2}-1}{\cos(x)-1}$

    (e) $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$

    (f) $\lim_{x\to\infty} \frac{x^3-2x+9}{x^2-9}$ This is top heavy vs bottom heavy type

    (g) $\lim_{x\to\infty} \frac{5x^2-2x+9}{7x^2-9}$ This is top heavy vs bottom heavy type

    (h) $\lim_{x\to\infty} \frac{x^3-2x+9}{3x^4-9}$ This is top heavy vs bottom heavy type

    (i) $\lim_{x\to\infty} \frac{\ln(x)}{3x^4-9}$ Back to L'Hopital's Rule

    (j) $\lim_{x\to\infty} \frac{e^x}{3x^4-9}$ Back to L'Hopital's Rule

    (k) $\lim_{x\to\infty} \frac{\ln(x)}{e^x}$

    (l) $\lim_{x\to\infty} \frac{\ln(x)}{\frac{1}{x}}$

    (m) $\lim_{x\to 0^+} \frac{\ln(1+x)}{\frac{1}{x}}$

    (n) $\lim_{n\to\infty} \frac{\ln(1+n^{-1})}{\frac{1}{n}}$

    (o) $\lim_{x\to 3} \frac{x^2-2x+9}{x^2-9}$

    (p) $\lim_{x\to 3} \frac{x^2-6x+9}{x^2-9}$

    (q) $\lim_{x\to 1^+} \frac{x-1}{\sqrt{x}-1}$

    (r) $\lim_{x\to\pi} \frac{\sin(x)}{x-\pi}$

    (s) $\lim_{x\to 0} \frac{4\sin(5x)}{3x}$

    (t) $\lim_{x\to 0} \frac{\sin(x)-x+\frac{1}{6}x^3}{x^5}$

    (u) $\lim_{x\to\infty} \frac{x^3+1}{e^x}$

    (v) $\lim_{x\to\infty} \frac{3x+\sqrt{x}+1}{x}$

    (w) $\lim_{x\to\infty} \frac{3x+x^{3/2}+1}{x}$

    (x) $\lim_{x\to 0} \frac{\tan(x)}{x}$

5 Always Be Computing Derivatives (ABCD)

11. Compute the following derivatives

   (a) $f(x) = 3\sec(x) - 5\ln(x) + 1$

   (b) $f(x) = \ln(x)$

   (c) $f(x) = 4\sin^2(x)\cos^2(x) + \cos^2(2x)$

   (d) $f(x) = x^2(\tan(x) + 1)^3$

   (e) $f(x) = \arctan(x) + e^{x^3}$

   (f) $f(x) = \arctan(x^3 + 1)$

   (g) $f(x) = \sin(x)\cos(x)$

   (h) $f(x) = x^x$

   (i) $f(x) = x^{x^x}$

6 Formulae and Rules

Know these by heart. Use flash cards or whatever helps you memorize them.

Formulae

0. $\frac{d}{dx}[k] =$

1. $\frac{d}{dx}[x^n] =$

2. $\frac{d}{dx}[e^x] =$

3. $\frac{d}{dx}[\ln x] =$

4. $\frac{d}{dx}[\sin x] =$

5. $\frac{d}{dx}[\cos x] =$

6. $\frac{d}{dx}[\tan x] =$

7. $\frac{d}{dx}[\cot x] =$

8. $\frac{d}{dx}[\sec x] =$

9. $\frac{d}{dx}[\csc x] =$

10. $\frac{d}{dx}\left[\sin^{-1} x\right] =$

11. $\frac{d}{dx}\left[\tan^{-1} x\right] =$

12. $\frac{d}{dx}\left[\sec^{-1} x\right] =$

The Power Rule

$$\frac{d}{dx}[(f(x))^n] = n(f(x))^{n-1}f'(x)$$

The Product Rule

$$\frac{d}{dx}\left[FS\right] = F'S + FS'$$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{N}{D}\right] = \frac{N'D - ND'}{D^2}$$

The Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$