MATH 2310 Practice Test 1 Answers

The follwing ae the answers without the work

1. Compute the limits

   (a) $\lim_{x\to 2} x^3 + 1 = 9$

   (b) $\lim_{x\to 2} \frac{x^2 - 2}{x^2 + 2} = \frac{2}{6}$

   (c) $\lim_{x\to 2} \frac{x^2 - 2x + 1}{x^2 - 2} = 1/2$

   (d) $\lim_{x\to 0} \frac{\sin(3x)}{3x} = 1$

   (e) $\lim_{x\to 0} \frac{\sin(3x)}{x} = 3$

   (f) $\lim_{x\to 0} \frac{\sin(x)}{3x} = \frac{1}{3}$

   (g) $\lim_{n\to\infty} \frac{4n^2 + 5}{3n^2 + 6n - 5} = 4/3$

   (h) $\lim_{n\to\infty} \frac{n^2}{n^3 + 1} = 0$

   (i) $\lim_{n\to\infty} \frac{n^3}{n^2 + 1} = \infty$

   (j) $\lim_{n\to -\infty} \frac{n^5}{n^3 + 1} = \infty$

   (k) $\lim_{n\to -\infty} \frac{n^3}{n^2 + 1} = -\infty$

   (l) $\lim_{n\to\infty} \sqrt{2n+1}\frac{3n + 2}{\sqrt{4n^3 + n + 11}} = \frac{3\sqrt{2}}{2}$

   (m) $\lim_{x\to 0^+} \frac{1}{3x} = +\infty$

   (n) $\lim_{x\to 0^-} \frac{1}{3x} = -\infty$

   (o) $\lim_{x\to 0} \frac{1}{3x} = \text{DNE}$

   (p) $\lim_{x\to 0^+} \frac{1}{x^2} = \infty$

(q) $\lim_{x\to 0^-} \frac{1}{x^2} = \infty$

(r) $\lim_{x\to 0} \frac{1}{x^2} = \infty$

(s) $\lim_{x\to 1} \frac{x+1}{x^2-1}$ DNE You must show the right handed and left handed limits

2. Compute the derivative using the **definition** of the derivative

   (a) $f(x) = 3x + 5$ at $x = -1$

   (b) $f(x) = x^2$ at $x = 2$

   (c) $f(x) = x^2$

   (d) $f(x) = 5x + 1$

   (e) $f(x) = \sqrt{x}$
   Solution:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - x}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h} - x}{h} \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\
&= \lim_{h\to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\
&= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
\end{aligned}
$$

   (f) $f(x) = \frac{1}{x}$

3. Compute the derivative from the formulae.

   (a) $f(x) = 3x + 5\sqrt{x} - \sin(x)$
   $f'(x) = 3 + 5/2x^{-1/2} - \cos(x)$

   (b) $f(x) = 5\cos(x) - 4\ln(x) + \sin(x)$
   $f'(x) = -5\sin(x) + 4\frac{1}{x} + \cos(x)$

   (c) $f(x) = \frac{1}{x} - \sqrt{x}$ $f'(x) = -x^{-2} - \frac{1}{2}x^{-1/2}$

(d) $f(x) = \frac{4-2x+2x^2}{\sqrt{x}}$
Since $f(x) = 4x^{-1/2} - 2x^{1/2} + 2x^{3/2}$ we have $f'(x) = -2x^{-3/2} - x^{-1/2} + 3x^{1/2}$

(e) $f(x) = \frac{4\sqrt{x}-2+x^{3/2}}{\sqrt{x}}$
Since $f(x) = 4 - 2x^{-1/2} + x$ we have $f'(x) = 0 + x^{1/2} + 1$

(f) $f(x) = \frac{4\sqrt{x}-2}{x^{3/2}}$
Since $f(x) = 4x^{-1} - 2x^{-3/2} + 2x$ we have $f'(x) = -4x^{-2} + 3x^{-5/2}$

(g) $f(x) = \sin(x) - 2e^x + \ln(x)$
$f'(x) = \cos(x) - 2e^x + \frac{1}{x}$

4. Compute the derivative from the formulae and use product rule and the power rule.

(a) $f(x) = 5\sqrt{x}\sin(x)$
$f'(x) = 5(1/2x^{-1/2})\sin(x) + 5\sqrt{x}\cos(x)$

(b) $f(x) = 5\cos(x)\sin(x)$
$f'(x) = -5\sin(x)\sin(x) + 5\cos(x)\cos(x)$

(c) $f(x) = 4\ln(x)\cos(x)$
$f'(x) = 4\frac{1}{x}\cos(x) - 4\ln(x)\sin(x)$

(d) $f(x) = \ln(x)$
$f'(x) = 1/x$

(e) $f(x) = (\ln(x) + 7x)^3$
$f'(x) = 3(\ln(x) + 7x)^2 \left(\frac{1}{x} + 7\right)$

(f) $f(x) = (3x + 2)^3$
$f'(x) = 3(3x + 2)^2(3)$

(g) $f(x) = (3x^3 + 2)^3$
$f'(x) = 3(3x^3 + 2)^2(9x^2 + 0)$

(h) $f(x) = \sqrt{3x^3 + 2}$
$f'(x) = \frac{1}{2}(3x^3 + 2)^{-1/2}(9x^2 + 0)$

(i) $f(x) = (x - 1)^4(3x + 2)^3$
$f'(x) = 4(x - 1)^3(1)(3x + 2)^3 + (x - 1)^4 3(3x + 2)^2(3)$

(j) $f(x) = e^x(1 + e^x)^{-2}$
$f'(x) = e^x(1 + e^x)^{-2} + e^x(-2)(1 + e^x)^{-3}(e^x)$

(k) $f(x) = (\sin(x))^{-1}$
$f'(x) = (-1)(\sin(x))^{-2}(\cos(x))$

(l) $f(x) = (\cos(x))^{-1}$
$f'(x) = (-1)(\cos(x))^{-2}(-\sin(x)) = \frac{\sin(x)}{\cos^2(x)}$