

2310

$$1(a) \lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

$$\frac{e^0 - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0} \rightarrow \text{MORE WORK (LH)}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{e^x}{1} = \frac{e^0}{1} = \boxed{1}$$

$$(b) \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x}$$

$$\frac{e^0 - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{2x e^{x^2} - 0}{1} = \frac{2(0) e^0}{1} = \boxed{0}$$

$$(c) \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{2x e^{x^2}}{2x} = \lim_{x \rightarrow 0} \frac{e^{x^2}}{1} = e^0 = 1$$

$$(d) \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{\sin(4x)}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{3e^{3x}}{4 \cos(4x)}$$

$$\frac{3e^0}{4 \cos(0)} = \boxed{\frac{3}{4}}$$

$$(e) \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x \sin(4x)}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin(4x) + x 4 \cos(4x)}$$

$$\frac{e^0 - 1}{\sin(0) + 0} = \frac{0}{0}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0} \frac{e^x}{4 \cos(4x) + 1(4 \cos(4x)) + x \cdot 16(-\sin(4x))}$$

$$= \boxed{\frac{1}{8}}$$

slight
edit
to
work sheet
here

$$(1k) \lim_{x \rightarrow 0^+} (1+3x^2)^{2/x^2}$$

$$\ln L = \lim_{x \rightarrow 0^+} \ln (1+3x^2)^{2/x^2}$$

$$= \lim_{x \rightarrow 0^+} \frac{2}{x^2} \ln(1+3x^2)$$

$$= \lim_{x \rightarrow 0^+} \frac{2 \ln(1+3x^2)}{x^2}$$

$$\stackrel{(L^+)}{=} \lim_{x \rightarrow 0^+} \frac{2 \cdot \frac{1}{1+3x^2} (6x)}{2x} = \lim_{x \rightarrow 0^+} \frac{12}{1+3x^2} = \frac{12}{2} = \boxed{6}$$

$$(1m) \lim_{x \rightarrow 0^+} x^x$$

$$\ln(L) = \lim_{x \rightarrow 0^+} \ln(x^x)$$

should look like 1g

$$= \lim_{x \rightarrow 0^+} x \ln(x) = \lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^{-1}} \stackrel{(L^+)}{=} \dots = 0$$

$$\text{So } \ln L = 0$$

$$\text{Then, } L = \boxed{e^0}$$

$$1(f) \lim_{n \rightarrow \infty} e^{-n} (n^2 + 1)$$

$$= \lim_{n \rightarrow \infty} \frac{n^2 + 1}{e^n}$$

$$\stackrel{\text{L'H}}{=} \lim_{n \rightarrow \infty} \frac{2n + 0}{e^n}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{e^n} = 0$$

$$e^{-\infty} (\infty^2 + 1)$$

$$0 \cdot \infty$$

INDET. $\Rightarrow$ TURN INTO
FRACTION, AND
USE L'H

$$1g) \lim_{x \rightarrow 0^+} x \ln(x)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^{-1}}$$

$$\frac{\ln(1-3x)}{x}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-x^{-2}} = \lim_{x \rightarrow 0^+} \frac{1}{x} (-x^2)$$

$$= \lim_{x \rightarrow 0^+} -x = 0$$

$$(1h) \lim_{x \rightarrow 0^+} \frac{\ln(1-3x)}{x}$$

$$\frac{\ln(1-0)}{0} = \frac{0}{0}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1-3x} (-3)}{1} = \boxed{-3}$$

$$(1i) \lim_{x \rightarrow 0^+} \frac{\ln(1-3x^2)}{4x^2}$$

$$\frac{(1-3x)^{1/4}}{(1+3x^2)^{2/4}}$$

$$\stackrel{(LH)}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1-3x^2} (-6x)}{8x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{1-3x^2} (-6)}{8} = \frac{\frac{1}{1-0} (-6)}{8} = \frac{-6}{8} = \boxed{\frac{-3}{4}}$$

$$(1j) \lim_{x \rightarrow 0^+} (1-3x)^{1/x}$$

1^∞ INDET.

use Log limit taking

$$\ln L = \lim_{x \rightarrow 0^+} \ln (1-3x)^{1/x}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x} \cdot \ln(1-3x)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(1-3x)}{x} \stackrel{(LH)}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1-3x} (-3)}{1} = \boxed{-3}$$

But we aren't done yet

$$\ln L = -3$$

so $\boxed{L = e^{-3}}$

$$(1 a) \lim_{x \rightarrow 0^+} [\sin(x)]^x$$

$$\ln(U) = \lim_{x \rightarrow 0^+} \ln(\sin(x))^x$$

$$= \lim_{x \rightarrow 0^+} x \ln(\sin(x))$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(\sin(x))}{x^{-1}} \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{\sin(x)} \cos(x)}{-x^{-2}} = \lim_{x \rightarrow 0^+} \frac{-x^2}{\sin(x)} \cos(x)$$

$$= \lim_{x \rightarrow 0^+} (-x) \cdot \frac{x}{\sin(x)} \cos(x)$$

$$= 0 \cdot 1 \cdot \cos(0) = 0$$

So $L = \boxed{e^0}$

$$(2 a) \int 7 \sec^2(x) - 3 \, dx = 7 \tan(x) - 3x + C$$

$$(b) \int 3x^3 - \sqrt{x} - \frac{3}{x} + \frac{1}{\sqrt{x}} \, dx$$

$$= \int 3x^3 - x^{1/2} - 3 \cdot \frac{1}{x} + x^{-1/2} \, dx$$

$$= \frac{3x^{3+1}}{3+1} - \frac{x^{1/2+1}}{1/2+1} + 3 \ln(x) + \frac{x^{-1/2+1}}{-1/2+1} + C$$

$$(2(c)) \int x(x^2-1) dx = \int x^3 - x dx$$

$$= \frac{x^4}{4} - \frac{x^2}{2} + C$$

$$2(d) \int \frac{11}{\sqrt{1-x^2}} - \frac{3}{x} dx = \int 11 \cdot \frac{1}{\sqrt{1-x^2}} - 3 \cdot \frac{1}{x} dx$$

$$= 11 \arcsin(x) - 3 \ln(x) + C$$

$$2(g) \int \sin(2x-3) dx$$

$$u = 2x-3$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$= \int \sin(u) \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \sin(u) du = -\frac{1}{2} \cos(u) + C$$

$$2(h) \int e^{x+2} dx$$

$$u = x+2$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$= \int e^u du$$

$$= e^u + C$$

$$= e^{x+2} + C$$

$$2(j) \int x \sqrt{1+x^2} dx$$

$$= \int (1+x^2)^{1/2} (x dx)$$

$$= \int (u)^{1/2} \frac{1}{2} du$$

$$= \frac{1}{2} \int u^{1/2} du = \frac{1}{2} \frac{u^{1/2+1}}{1/2+1} + C = \frac{1}{2} \frac{(1+x^2)^{3/2}}{\frac{3}{2}} + C$$

$$u = 1+x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$2(l) \int x \csc^2(x^2+1) dx$$

$$= \int \csc^2(u) \left(\frac{1}{2} du \right)$$

$$= \frac{1}{2} \int \csc^2(u) du$$

$$= -\frac{1}{2} \cot(u) + C = \boxed{-\frac{1}{2} \cot(x^2+1) + C}$$

$$u = x^2+1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$2(m) \quad \text{try} \quad u = e^x + 1$$

$$2(n) \quad \text{try} \quad u = \sin(x) + 1$$

3(a) $f(x) = 3x + 5$ at $x = -1$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 3x + 5$$

$$f(-1) = 3(-1) + 5 = 2$$

$$f'(-1) = \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h}$$

$$f(-1+h) = 3(-1+h) + 5$$

$$= \lim_{h \rightarrow 0} \frac{[3(-1+h) + 5] - [2]}{h} = \lim_{h \rightarrow 0} \frac{-3 + 3h + 5 - 2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h} = \lim_{h \rightarrow 0} 3 = \boxed{3}$$

(c) $f(x) = x^2$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h = \boxed{2x}$$

3e) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

how do we work with radicals?
use the conjugate!

$$= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \boxed{\frac{1}{\sqrt{x+0} + \sqrt{x}}}$$

(4a) $x^3 + y^3 = 2x + 5$

$$3x^2 + 3y^2 \frac{dy}{dx} = 2$$

$$\frac{3y^2 \frac{dy}{dx}}{3y^2} = \frac{2 - 3x^2}{3y^2} \Rightarrow$$

$$\boxed{\frac{dy}{dx} = \frac{2 - 3x^2}{3y^2}}$$

(4b) $x^3 y^3 = 2x + 5$

$$3x^2 y^3 + x^3 3y^2 \frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{2 - 3x^2 y^3}{3x^3 y^2}$$

4e) $\sin(xy^2) = 2x + 5y - 7$

$$\cos(xy^2) \left[(1)y^2 + x \cdot 2y \frac{dy}{dx} \right] = 2 + 5 \frac{dy}{dx}$$

$$y^2 \cos(xy^2) + 2xy \frac{dy}{dx} \cos(xy^2) = 2 + 5 \frac{dy}{dx}$$

Factor out $\frac{dy}{dx}$ and then

$$5 \frac{dy}{dx} + 2xy \frac{dy}{dx} \cos(xy^2) = 2 - y^2 \cos(xy^2)$$

$$\text{so } \boxed{\frac{dy}{dx} = \frac{2 - y^2 \cos(xy^2)}{5 + 2xy \cos(xy^2)}}$$

$$(4h) \quad y = x^{x^2}$$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

$$\frac{1}{y} y' = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x}$$

$$y' = y [2x \ln(x) + x]$$

$$y' = x^{x^2} [2x \ln(x) + x]$$

$$(5) \quad s(t) = -4.9t^2 + 3t + 1$$

First WE find the velocity & acceleration functions

$$v(t) = s'(t) = -9.8t + 3$$

$$a(t) = s''(t) = -9.8$$

$$(a) \quad s(0) = -4.9(0)^2 + 3(0) + 1 = 1 \text{ m}$$

$$(b) \quad v(0) = -9.8(0) + 3 = 3 \text{ m/s}$$

$$(c) \quad v(t) = 0$$

$$-9.8t + 3 = 0$$

$$t = \frac{3}{9.8} \text{ sec}$$

$$(d) \quad s(t) = 0$$

$$-4.9t^2 + 3t + 1 = 0$$

$$t = \frac{-3 \pm \sqrt{9 - 4(-4.9)(1)}}{2(-4.9)}$$

$$= \frac{-3 \pm \sqrt{28.6}}{-9.8}$$

$$t = \frac{-3 - \sqrt{28.6}}{-9.8}$$

$$\text{or } t = \frac{-3 + \sqrt{28.6}}{-9.8}$$

Not

$$(e) \quad v\left(\frac{-3 - \sqrt{28.6}}{-9.8}\right) = -9.8\left(\frac{-3 - \sqrt{28.6}}{-9.8}\right) + 3$$

$$= \sqrt{28.6} \text{ m/sec}$$

(f) I did it!