Math 3160 - Test 2

Name:____

1. Find standard equation for the plane (in $\mathbb{R}^3$) so that

   (a) the plane contains the point $P(2,2,-1)$, $Q(1,0,3)$ and $R(0,-1,0)$.

   (b) the plane contains the the point $P(2,2,-1)$ and is perpendicular to the vector $(1,-2,0)$.

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2. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } x + 3z = 0\}$. Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

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3. Let $S = \{(4,2,0),(1,1,1),(1,1,0)\}$.

   (a) Is S linearlly independent?

   (b) Is $(2,2,2) \in \mathrm{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2)$?

   (c) Does S span $\mathbb{R}^3$?

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4. Let $B = \{(1,0),(0,1)\}$, $B_1 = \{(1,2),(1,3)\}$ and $B_2 = \{(1,-1),(2,1)\}$.

   (a) Find the change of basis matrices for $P_{B\to B_1}$ and $P_{B_1\to B_2}$.

   (b) Find the coordinates of the point $(1,3)$ (given in the standard basis) relative to the bases B_1 and B_2.

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5. Write the matrix for the following transformations described below.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 45° counter-clockwise.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis.

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis and then the plane is rotated by 45° counter-clockwise.

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6. The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is given by the formula
$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x - y + 2w \\ x - 2y + 3w \\ y - w \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear trasformation T.

(b) Compute the basis for the Range of T, which is the Column Space of A.

(c) Find a basis for the null space of A, NULL(A).

(d) Compute the dimension of COL(A) and NULL(A). The dimension of the range of T is called the rank of T and the dimension of the null space is called the nullity.

(e) What is the dimension of the domain of T and the codomain of T? Again, compare Rank, Nullity and the dimension of the Domain. Do you see a relation?

7. For the following matrices find the charactersitic equation, the eigenvalues and their cooresponding eigen vectors.

$$A = \begin{bmatrix} 5 & 3 \\ -3 & -1 \end{bmatrix}$$

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8. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a vector space V with the following properties:

   - $\mathbf{v}_1$ is perpandicular to $\mathbf{v}_2$ and $\mathbf{v}_3$, and $\mathbf{v}_2$ is perpandicular to $\mathbf{v}_3$
   - the norm of each vector is 1.

   (a) What is $\mathbf{v}_2 \cdot \mathbf{v}_3$?

   (b) What is the norm of $\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3$?

   (c) What is the angle between $\mathbf{v}_1 + 3\mathbf{v}_2 - 2\mathbf{v}_3$ and $\mathbf{v}_2$?

   (d) What is the norm of $a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$ where $a, b, c \in \mathbb{R}$?
