Math 3160 - Quiz 8

Name:____

1 Orthogonalization

1. Let $\mathbf{v} = (1,2,3)$ and $\mathbf{w} = (-1,0,1)$ Compute $Proj_{\mathbf{v}}\mathbf{w}$ and $Proj_{\mathbf{w}}\mathbf{v}$.
2. wedge problem like in class
3. Let $\mathbf{v} = (1,2,3)$ and $\mathbf{w} = (0,1,-1)$ Find the component of $\mathbf{v}$ parallel to $\mathbf{w}$ and the perpandicular component. We called these $\mathbf{C}_1$ and $\mathbf{C}_2$ respectively.
4. Let $S = \{(1,0,1), (-1,1,1), (0,-1,1)\}$. Is S orthonormal? Show this.
5. Let $S = \{(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{2}, \frac{1}{2}, -\frac{1}{\sqrt{2}})\}$. Is S orthonormal? Show this.
6. Note $\mathcal{B} = \{\mathbf{v}_1 = (\frac{4}{5}, \frac{3}{5}), \mathbf{v}_2 = (\frac{3}{5}, -\frac{4}{5})\}$ is an orthonormal basis for $\mathbb{R}^2$.
   (a) Compute $P_{S\to\mathcal{B}}$ where S is the standard unit normal basis.
   (b) Use Problem 6a to express the vector $\mathbf{v} = (-3,2)$ relative to the basis $\mathcal{B}$.
   (c) Compute $\mathbf{v}\cdot\mathbf{v}_1$ and $\mathbf{v}\cdot\mathbf{v}_2$.
   (d) Use Problem 6c to express the vector $\mathbf{v} = (-3,2)$ relative to the basis $\mathcal{B}$.
7. Let $S = \{(1,1,1), (0,1,1), (0,0,1)\}$.
   (a) Show S is a basis.
   (b) Orthogonalize it.
   (c) Write $(3,-1,2)$ in the terms of the basis.
8. Let $S = \{(1,2,3), (-1,1,0), (0,0,1)\}$.
   (a) Show S is a basis.
   (b) Orthogonalize it.

Gram-Schmidt Orthoganlization

To convert a basis $\{u_1, u_2, \ldots, u_r\}$ into an orthogonal basis $\{v_1, v_2, \ldots, v_r\}$, perform the following computations:

- $\mathbf{v}_1 = \mathbf{u}_1$
- $\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1$
- $\mathbf{v}_3 = \mathbf{u}_3 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2$

To convert to an orthonormal basis, normalize each vector.