Math 3160 - Quiz 4

Name:_____

1. Prove Theorem 4.1.1
2. Show that $W = \{(a, b, a + 2b) | a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.
3. Show that $W = \{(a + 1, 3a, a + b) | a, b \in \mathbb{R}\}$ is **not** a subspace of $\mathbb{R}^3$.
4. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.
   (a) Write down the definition of a **linear transformation**.
   (b) For the set Range defined below prove Range is a subspace of R^m. Define the set
   $$\text{Range} = \{T(\mathbf{v}) | \mathbf{v} \in \mathbb{R}^n\}$$
5. Write down two equivalent definitions for a list of vectors to be **independent**.
6. Show each of the following lists are independent or dependent
   (a) $(1, 2, 3)$, $(1, 0, -1)$ in $\mathbb{R}^3$
   (b) $(1, 2, 3)$, $(4, 5, 6)$, $(7, 8, 9)$ in $\mathbb{R}^3$
   (c) x, $1 + x$, $1 - x$ in $\mathcal{P}_2$
7. Write down the definition for a **basis** of a vector space. Write down the definition for **span**.
8. Are $v = (-1, 6, 5, -2)$ or $v = (1, 1, 0, 1)$ in the span of the following set of vectors?
   $$\{(1, 2, 0, 1), (1, 0, -1, 2), (0, 1, 1, 1), (1, 7, 3, 0)\}$$
9. Show the following are or are not a basis for the given vector space. Explain why or why not it is a basis.
   (a) $(1, 2, 3), (4, 5, 6), (2, 3, 1)$ in $\mathbb{R}^3$
   (b) $(1, 2, 3), (4, 5, 6), (7, 8, 9)$ in $\mathbb{R}^3$
   (c) $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in $\mathbb{R}^3$
   (d) $(1, 0, 1), (0, 1, 2)$ in $W = \{(a, b, a + 2b) | a, b \in \mathbb{R}\}$