MA 2320: Quiz 6

Name:_____

1. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2+1}{2k^2-1}\right)^k$
2. $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{2}{k}\right)^{k^2}$
3. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^3+1}$
4. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{\sqrt{k}-1}$
5. Find the Interval of Convergence for
   (a) $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^k\sqrt{k}}x^n$
   (b) $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}x^n$
   (c) $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{n!}x^n$
6. $f(x)=\ln(x)$
   (a) Find the Taylor Series for $f(x)$ with $c=1$ from the **definition**.
   (b) Find the Interval of convergence for this series.
   (c) Use $T_3(x)$ to approximate $\ln(1.1)$. Find the remainder using Taylor remainder formula to compute the accuracy of your approximation.
7. $f(x)=xe^{x^2}$
   (a) Find the Taylor Series for $f(x)$ with $c=0$ **using known Taylor series**.
   (b) Find the Interval of convergence for this series.
   (c) Use $T_3(x)$ to approximate $\ln(1.1)$. Find the remainder using Taylor remainder formula to compute the accuracy of your approximation.
8. Find the Taylor Series for $f(x)=\frac{\sin(x^2)-x^2}{x^2}$ with $c=0$ **using known Taylor series**.
9. Use the series from Problem 8 to compute the following limit.

$$\lim_{x\to 0}\frac{\sin(x^2)-x^2}{x^2}$$

10. Find the Taylor Series for $f(x) = \arctan(x)$ with $c = 0$ **using known Taylor series**.

11. Use the series from Problem 10 to get a series representation for $4\arctan(1)$. Why is this series interesting?

12. Compute the following integrals:

    (a) $\displaystyle\int xe^{x^2}\,dx$

    (b) $\displaystyle\int x^2e^x\,dx$

    (c) $\displaystyle\int \frac{2x^2-1}{x^3-x^2}\,dx$