Math 6250: Final Review

Review Test 1 review with special attention to the problems after number 24, this worksheet and the one from class.

1. Find the symmetric decomposition for the following functions over the given ω.

   (a) $f(x) = x^2 - x + 1$ where $\omega = -1$.

   (b) $f(x) = x^2 - x + 1$ where $\omega = i$.

   (c) $f(x) = e^x$ where $\omega = -1$. Do you recognize these functions?

   (d) $f(x) = e^x$ where $\omega = i$.

   where we saw
   $$f_j = \sum_{k=0}^{n-1} \frac{1}{\omega^{jk}} f(\omega^k x)$$
   and
   $$f = \sum_{k=0}^{n-1} f_j.$$

2. Compute the following Taylor series from the formula at the indicated point.

   (a) $f(x) = \sin(3x)$ at $c = 0$

   (b) $f(x) = \sin(3x)$ at $c = \pi/2$

   (c) $f(x) = x^2 - 3$ at $c = 1$

   (d) $f(x) = e^{x/2}$ at $c = 1$

3. Compute the following Taylor series at $c = 0$ from know series using algebra and calculus.

   (a) $f(x) = \frac{\sin(x)}{x}$.

   (b) $f(x) = \ln(1 + x)$

   (c) $f(x) = \arctan(x)$

   (d) $f(x) = \frac{\cos(x^2)-1}{x^2}$

   (e) $f(x) = e^{x^2}$

4. Use the results from Problem 3 to find

(a) a series representation of π

(b) The sum of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$

(c) a series representation of $e^{1/2}$

(d) a series representation of $\int_{-1}^{1} e^{-x^2}\, dx$

5. Prove that

(a) $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges.

(b) $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$.

6. State the definition of the Arithmetic (AM), Geometric (GM) and Harmonic (HM) means. How are the three means related?

7. What is the Cauchy inequality? Use it to help you prove the inequality from Problem 6.

8. Using the inequality from Problem 6 solve the following.

(a) Prove that if the product of n positive real numbers is 1, then their sum is at least n.

(b) Let $(a_k)_{k=1}^{n}$ be a sequence of possitive reals. And let $(b_k)_{k=1}^{n}$ be a permutation of $(a_k)_{k=1}^{n}$. Then

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \cdots + \frac{a_n}{b_n} \geq n$$

(c) Show that $(1 + a^2)(1 + b^2) \geq 4ab$ for all $a, b \in \mathbb{R}$.

(d) Let $x, y, z \in \mathbb{R}^+$ so that $xyz = 32$. Show

$$x^2 + 4xy + 4y^2 + 2z^2 \geq 96.$$

(e) When I drive to school in the moning I take the LIE for the first half and drive at 60 mph and take Route 25 for the second half of the trip and drive at 30 mph. What is my average speed? Note I am confused by what “half” means in this problem. Does it mean for half of the time or half of the distance. Compute both answers.