Math 4160 - Test 1

Name:____

1. Let $V = \{(x, y, z) \in \mathbb{R}^3 | x \neq 0 \text{ and } z \neq 0\}$. And define the two operations

   $\oplus$: $(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1x_2, x_1y_2 + y_1z_2, z_1z_2)$

   $\odot$: $k \odot (x_1, y_1, z_1) = (kx_1, ky_1, kz_1)$

   (a) Compute $(0, 4, 4) \oplus (-1, 2, 3)$ and compute $2 \odot (1, 1, 0)$.

   (b) Show $\mathbf{0} \neq (0, 0, 0)$.

   (c) Show $\mathbf{0} = (1, 0, 1)$.

2. Let $V = \{(x, y, z) \in \mathbb{R}^3 | x \neq 0 \text{ and } z \neq 0\}$. And define the two operations

$$\oplus\colon \quad (x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 x_2, x_1 y_2 + y_1 z_2, z_1 z_2)$$
$$\odot\colon \quad k \odot (x_1, y_1, z_1) = (kx_1, ky_1, kz_1)$$

(a) Prove that V is closed under $\oplus$.

(b) Prove that V is not closed under $\odot$.

3. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } 2x + y = 0\}$. Use the two step subspace test to show $(W, +, \cdot)$ is a subspace of $\mathbb{R}^3$.

4. Let $S = \{x, x+2, x^2 - x - 1, x^2\}$ be a set in P_2. Show S linearly dependent by finding a nontrivial linear combination of the elements of S equal to zero.

5. The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is given by the formula
$$T(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = \begin{bmatrix} x - 2w \\ w + z \\ x - 3z \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear transformation T.

(b) Compute the basis for the Range of T.

(c) Find a basis for NULL(A).

(d) Compute and compare Rank, Nullity and the dimension of the Domain.

6. Let $B_1 = \{(0,1),(2,1)\}$, $B_2 = \{(2,-1),(2,0)\}$ and let B be the standard unit basis for $\mathbb{R}^2$.

   (a) Find the change of basis matrices for $P_{B_1 \to B_2}$ and $P_{B_1 \to B_2}$.

   (b) The point $(4,6)_{B_1} = 4(0,1) + 6(2,1) = (12,10)$ where (12,10 is given in the standard basis. Find the coordinates of $(4,6)_{B_1}$ relative to the basis B_2.

7. Write the matrix for the following transformations described below.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 30° counter-clockwise.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis.

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 30° counter-clockwise and then reflected about the x-axis.

   (d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis and then rotated by 30° counter-clockwise.